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Abstract

In this paper, we prove that the noncooperative solution with mutually consistent conjectures is the same as the contract curve of the cooperative solution in a two-person game. A numerical example is also presented.

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1. Introduction

Economic conflict, or competitive behaviour between a finite and possibly small number of agents, is best modelled as a game. There are obvious applications in oligopoly theory (Friedman, 1977, 1983), international policy coordination (Oudiz and Sachs, 1985), wage bargaining (Calmfors and Drifill, 1988), environmental issues (Pethig, 1992), development economics (Li and Ma, 1996) as well as in many other areas.

In two-person games, two distinct behaviour patterns are possible: cooperative vs. non-cooperative. In a non-cooperative game (e.g. Nash or conjectural variations, see Basar and Olsder, 1982), each player will try to minimise his own cost function, conditional on the opponent's decision rule. But it is known that such non-cooperative decisions are socially inefficient (Da Cuhna and Polak, 1967). In contrast, a cooperative game is to minimise a joint cost function by the two players working together and will generate Pareto optimal outcomes.
In this paper, we show that, under certain conditions, non-cooperative behaviour can turn out to be Pareto efficient as well, i.e. the players in a non-cooperative game can behave as if they were playing a cooperative game, even though there is no pre-commitment for them to do so. That implies a built-in self-enforcement mechanism which would lead them to behave cooperatively out of self-interest. To show this, we work with one particular form of non-cooperative game: the conjectural variations equilibrium. Our result therefore extends earlier work by De Bruiyne (1979), which showed that the Nash noncooperative equilibrium is Pareto optimal only when the targets and preferences are identical for all players.

2. **Conjectural Variations Solutions**

Non-cooperation implies that each player in the game maximises his self-interest, subject to his perception of the constraints on his decision variables, conditional on rational expectations about the decisions taken by others. For simplicity, we shall restrict attention to two-player static games here. Each decision maker has one
instrument, $x_i$ to reach one target $y_i$ ($i=1,2$). The private interests of each player can then be represented by the cost functions:

$$w_i = w_i( y_i, x_i ), \ i = 1, 2.$$ 

The players face a static reduced-form economic system:

$$y_i = f_i( x_1, x_2 ), \ i = 1, 2.$$ 

Given a suitable starting point, conjectural variations can now be used to deduce appropriate reaction terms within the optimisation process. Like in chess, each player recognises that his decisions affect the targets and hence the policy choice of his opponent, and that variations in the latter then influence his targets again and hence what he (the first player) should decide to do. Instead of optimising conditional on fixed reactions by the opponent ('given he does that, what should I do?'), each player must now anticipate his opponent's reactions ('if I do this, how will he react and how should I best accommodate that?, etc.'). The conjectural variations employed by each player here are captured by $\partial x_1/\partial x_2$ and $\partial x_2/\partial x_1$ (while the Nash equilibrium, of course, sets $\partial x_1/\partial x_2=\partial x_2/\partial x_1=0$ ex ante). By optimising the decisions associated with each variation in the
conjectured responses to the currently envisaged choices, and then
constraining those variations so that the new pair of decisions
represent an improvement over the old pair, we reach the closed-loop
conjectural variations equilibrium. That equilibrium holds only
when both players perceive that no further gains can be made by
varying their reactions to the decisions currently expected from their
opponent, because to do so would trigger counter-reactions (in the
opponent's interest) which would more than offset the gains of
unilateral action.

There has been extensive discussion of the interpretation of
conjectural variations model (see, for example, Bresnahan, 1981;
Kamien and Schwartz, 1983). It has been argued by Friedman
(1983), among others, that it arises in a static one-shot game, cannot
be taken as a literal expectations of future reactions. Many game
theorists prefer to utilise repeated game models, which, however,
have their own weaknesses. A paper by Kalai and Stanford (1985)
explicitly analyzes conjectural variations in a repeated game
framework, showing that various conjectural variations can be maintained as credible equilibria. While Brander and Spencer (1985) and Dixit (1988) hold the view that the conjectural variations are simply a useful and intuitive summary measure of market conduct.

Recent applications of conjectural variations model in microeconomics can be found in the papers by Brander and Zhang (1989, 1993) who assessed the post-deregulation competitiveness of the US airline industry. Applications of conjectural variations model in the international macroeconomics can be found in Sterdyniak and Villa (1993) who investigated the international policy coordination issues.

However, the conjectural variations equilibrium is indeterminate as the conjectures are formed outside the model. Independent efforts have been made to narrow the possible market equilibria by imposing additional constraints that the conjectural variations be 'consistent' (Friedman, 1977; Laitner, 1980; Bresnahan, 1981; Kamien and Schwartz, 1983). One way to solve this multiply solution problem
is by imposing self consistent conjectures for the players (see, for example, Bresnahan, 1981): the conjecture in the reaction function is set to the slope of the reaction function. Under certain conditions, the self consistent conjectures equilibrium is unique. In this paper, we try to examine the conjectural variations solution by imposing an alternative assumption: the conjectures in the reaction function of each player are mutually consistent. We find that this assumption will give us a general condition under which the conjectural variations solution becomes a cooperative solution.

3. The Equivalence of Noncooperative Solution with Mutually Consistent Conjectures and Cooperative Solution

The necessary conditions for optimal decisions with conjectural variations are

\[ E_i(\frac{\partial x_j}{\partial x_i}) \]  \hspace{1cm} (1)

for i=1 and 2 (and i≠j), where \( E_i(\frac{\partial x_j}{\partial x_i}) \) is the player i's expectation.
on the player j’s response. From this we derive:

**Theorem** Suppose player i (i=1,2) tries to minimise his objective function \( w_i \) by choosing instrument \( x_i \) subject to a constraint \( f_i \):

Then, the conjectural variations general solution is the same as the contract curve, if the two players' expectations are mutually consistent, i.e.

\[
E_i(\partial x_j/\partial x_i) = \partial x_j/\partial x_i
\]  

(2)

where \( E_i(.) \) is player i's expectation (i=1,2 and i\( \neq \)j). This states that the reaction player j chooses is consistent with what player i expect him to choose.

**Proof**

For any given value of objective function \( k \), the indifference curve for player i is:
Therefore, the derivative of the indifference curve for player 1 is:

Similarly, the derivative of the indifference curve for player 2 is

The contract curve is the locus at which the derivatives of the two indifference curves are the same, i.e. the locus of the tangent points of the two players' indifference curves:

For the conjectural variations game in (1), player 1's decisions must satisfy:

where $E_1(\partial x_2/\partial x_1) \neq 0$, and player 2's decisions
where $E_2(\partial x_1/\partial x_2) \neq 0$. But in equilibrium we are able to cancel the terms of $E_2(\partial x_1/\partial x_2)$ and $E_1(\partial x_2/\partial x_1)$ under mutually consistent expectations condition, since:

$$\text{Hence, in equilibrium we have:}$$

which is the same as the contract curve (6). That implies that there is an one-to-one correspondence between the contract curve and the mutually consistent conjectures solution.

[Q.E.D.]

To illustrate the theorem, we apply it to solve a particular problem which is given as follows:
The conjectural variations solution is:

\[ \frac{\partial w_1}{\partial x_1} = 0 \], i.e. \( y_1 \left[ 1 + E_1(\frac{\partial x_2}{\partial x_1}) \right] + x_1 = 0 \),

and

\[ \frac{\partial w_2}{\partial x_2} = 0 \], i.e. \( y_2 \left[ 1 + E_2(\frac{\partial x_1}{\partial x_2}) \right] + x_2 = 0 \),

implying that the noncooperative solution with mutually consistent conjectures must satisfy:

\[ x_1^2 + x_2^2 + 3x_1x_2 - x_1 - x_2 = 0 \]  \hspace{1cm} (13)

Meanwhile the indifference curves of the two players are, respectively:

\( (x_1 + x_2 - 1)^2 + x_2 = k_1 \) and \( (x_1 + x_2 - 1)^2 + x_2 = k_2 \) \hspace{1cm} (for any given \( k_1, k_2 \))

The derivatives of the indifference curves are:

and the contract curve of the cooperative solution is

\[ x_1^2 + x_2^2 + 3x_1x_2 - x_1 - x_2 = 0 \]  \hspace{1cm} (15)
which is exactly the same as the noncooperative solution with mutually consistent conjectures (13) (see Figure 1).

4. Conclusion

In this paper, we proved that the noncooperative solution with the mutually consistent conjectures is the same as the contract curve of the cooperative solution in the two-person game. This implies that the players in a non-cooperative game can sometimes behave like in a cooperative game, for their own self-interest, although there is no binding commitment for them to do so. The result clearly is important for the policymakers when they design the coordinated policy packages, for the study of the market behaviour and in other areas.

References


Bresnahan, T. F. (1981) Duopoly Models with Consistent Conjectures,


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Fig 1.

x2: Player 2

Noncoop sol'n

Coop sol'n

x1: Player 1