The Brain Drain, “Educated Unemployment,”
Human Capital Formation, and Economic Betterment

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Abstract

Extending both the “harmful brain drain” literature and the “beneficial brain gain” literature, this paper analyzes both the negative and the positive impact of migration by skilled individuals in a unified framework. The paper extends the received literature on the “harmful brain drain” by showing that in the short run, international migration can result in “educated unemployment” and in overeducation in developing countries, as well as in a brain drain from these countries. A simulation suggests that the costs of the negative consequences of “educated unemployment” and overeducation can amount to significant losses for the individuals concerned, who may constitute a substantial proportion of the educated individuals. Adopting a dynamic framework, it is then shown that due to the positive externality of the prevailing, economy-wide endowment of human capital on the formation of human capital, a relaxation in migration policy in both the current period and the preceding period can facilitate “take-off” of a developing country in the current period. Thus, it is suggested that while a controlled migration of skilled individuals may reduce the social welfare of those who stay behind in the short run, it improves it in the long run.

JEL Classification: F22; I30; J24; J61; J64; O15

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1. Introduction

Labor migration has long been a topic of intense interest in population research in general and in development economics in particular. The topic has been gaining added appeal in the era of globalization. The received wisdom has been that such migration results in a detrimental brain drain for the developing countries (for a systematic review see Bhagwati and Wilson 1989). A recent and growing literature argues that the brain drain is accompanied by a beneficial brain gain. The new writings contend that compared with a closed economy, an economy open to migration differs not only in the opportunities that workers face but also in the structure of the incentives that they confront; higher prospective returns to human capital in a foreign country impinge favourably on human capital formation decisions at home.

We seek to synthesize and extend the two strands of the received literature, and to analyze both the positive and the negative impact of migration in a unified framework. The basic analytical construct of this paper is delineated in Fan and Stark (2007), who show that the prospect of international migration results not only in a brain drain but also in “educated unemployment,” which is an important feature of the labor market in many developing countries. In the present paper we conduct our analysis in the framework of a “threshold externality” of human capital, which enables us to analyze the negative and the positive impact of migration in different periods, and to make welfare comparisons.

We extend the received literature of “harmful brain drain” by showing that in the short run international migration can result in “educated unemployment” and in overeducation, as well as in a brain drain. Specifically, in contrast with the literature that views the brain drain as the only negative consequence of international migration of skilled workers, we identify three possible negative short-run consequences. First, consistent with the “traditional”

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1 As noted by Stark (2004), this view has become so entrenched that it is regularly echoed by the informed press.


3 See, for example, the empirical observations with regard to “educated unemployment” in Fan and Stark (2007).
view, migration leads to a reduction in the “stock” of better-educated individuals, which in
turn reduces the average income in the developing country. Second, since some educated
individuals who would otherwise have taken jobs are lured into further education only to end
up unemployed, output shrinks. Third, since the possibility of migration induces individuals
in a developing country to acquire higher education, when some of these individuals end
up remaining in the country, the returns from their education could be less than its costs.
From their perspective, they are overeducated. If the country’s economy cannot “take off,”
then these individuals’ overeducation is socially inefficient (in the short run). Moreover,
the simulation shows that the costs of the two new negative consequences of migration
introduced in this paper, namely “educated unemployment” and overeducation, can amount
to significant losses for the individuals affected, who may constitute a substantial proportion
of the educated individuals. In addition, in per capita terms, the cost of a brain drain can
be relatively small if the proportion of the educated individuals in the economy is small.

However, we next demonstrate that in the long run (one generation down the road),
the legacy of a relaxation in migration policy prompts “take-off” of the economy. Drawing
on the studies by Azariadis and Drazen (1990) and Galor and Stark (1994) that link the
long-run growth in a country’s output with the average level of the country’s human capital,
we emphasize the role of a “threshold externality” of human capital in economic develop-
ment. (Azariadis and Drazen (1990) and Pritchett (1997) provide evidence in support of
the assumption of a threshold externality. Fan (2004) offers an explanation for the existence
of the threshold externality in economic development by showing that a poor economy will
engage in international trade, which will accelerate its growth, if and only if its average
level of human capital is sufficiently high.) In a dynamic framework we show that the brain
drain is accompanied by a “brain gain;” that the ensuing “brain gain” can result in a higher
average level of human capital in the home country; that the higher average level of human
capital can prompt “take-off” of the economy; and that the “take-off” can bite into the

There is an interesting literature on “overeducation” which in labor economics is defined somewhat
differently than in our setting (Sicherman, 1991). Interestingly, using American data, Sicherman shows
that overeducation can be partly explained by the mobility patterns of educated workers. In our setting,
overeducation is explained by the migration prospect of educated workers.
unemployment rate. In such a setting, overeducation can become dynamically efficient (due to the intergenerational externality effect of human capital) even though it may be statically inefficient. Thus, we depict a setting in which rather than being to blame for human capital drain and output contraction, the migration of educated workers is the harbinger of human capital gain and output growth. An analysis of the entire dynamics associated with the response of educated workers to the prospect of migration therefore raises the intriguing possibility that what at first sight appears to constitute a curse is, in fact, a blessing in disguise. Our results are more dramatic than those reported in the received literature because in our present framework the prospect of migration is taken to entail both depletion of human capital and unemployment of human capital, which stacks the cards more firmly against viewing migration as a catalyst for growth.

Our analytical predictions appear to be in line with some empirical observations. For example, from 1960 to 1980, countries characterized by high rates of migration of skilled labor (such as India and Ireland) were among those countries that experienced the lowest rates of economic growth (Summers and Heston, 1991). However, since the late 1980s (that is, after approximately one generation), both India and Ireland have experienced rapid economic growth, which to a large extent has been due to an expansion of their skill-intensive information technology sector. Thus, by analyzing and synthesizing both the “traditional” and the new views of migration, we present a framework that is in line with some intertemporal evidence.

In Section 2 we set up the basic analytical framework. Section 3 investigates the negative and the positive impact of migration in the framework of a “threshold externality” of human capital and of a rational-expectations equilibrium. Section 4 contains the welfare analysis. Section 5 presents simulations aimed at illustrating how the channels described in the model in the preceding sections could operate in reality. Section 6 offers conclusions.

\footnote{See, for example, Kapur and McHale (2003) on the link between migration and the growth of the IT sector in these countries.}
2. The basic analytical framework

The basic analytical framework of this paper draws on Fan and Stark (2007). Consider a world that consists of two countries: Home, H, and Foreign, F. Country H is developing and is poorer than the developed country F. Due to a policy of selective migration by F, only educated individuals (say university graduates) of H have a chance of working in, hence migrating to, F. An educated individual makes decisions in (at most) three stages. (1) When an individual graduates from a university, the individual participates in a draw that results in probable work in F. If the individual obtains a winning ticket, his income will be a constant $w^f$. The probability of being selected into work in F is $p$. (2) An individual who graduates and fails to secure work in F faces the following choices: to work or to wait for another draw. For example, if the individual were to work, little time (and energy) would be available for preparing applications and, in addition, the individual’s academic qualifications could depreciate, thereby lowering the probability of being picked up for work in F. For simplicity, it is assumed that if the individual works, he cannot participate in any additional draw so that his probability of ending up working in F is zero. If the individual does not work and awaits another draw, his chance of going abroad is $p'$, where

$$p' = p(1 + \alpha)$$

and $\alpha$ is a fixed parameter. To ensure that $0 < p' < 1$, we assume that $-1 < \alpha < \frac{1}{p} - 1$. (3) If an individual wins this draw, he will go abroad. Otherwise, he will work at home, receiving the home country’s mean wage rate.

The job offers in the second and the third stage follow an independently identical distribution. The cumulative distribution function of the wage offer, $\tilde{w}$, is $F(\bullet)$. We assume that $F(\bullet)$ is differentiable, and that

$$\tilde{w} \in [w^f, w^h]$$

and that the density function, $\frac{dF(w)}{dw} \equiv F'(w)$, is strictly positive in its domain. The expected income of the (risk-neutral) individuals in the third stage is

$$(1 - p')\bar{w} + p' w^f$$

(2.1)
where $\bar{w}$ is the mean wage in $H$, namely, 

$$\bar{w} = \int_{w_l}^{w_H} w dF(w).$$

In the second stage, if the individual receives a wage offer $w$ at $H$, he will accept it if and only if 

$$w > \frac{1}{1 + r}[(1 - p')\bar{w} + p'w^f],$$

where $r$ is the individual’s discount rate.

We define

$$w^c = \frac{1}{1 + r}[(1 - p')\bar{w} + p'w^f].$$

Then, the individual will accept the wage offer at $H$ if and only if 

$$w > w^c.$$

Further simplifying, we assume that

$$w^f \geq \frac{1}{1 + r} \bar{w};$$

“educated unemployment” will not exist in the absence of an additional possibility of migration (that is, when $p' = 0$). Clearly, (2.4) will be satisfied if $r$ is large enough.

Then, the fraction of the educated who are unemployed is

$$u \equiv P(\bar{w} \leq w^c) = F(w^c).$$

Clearly,

$$\frac{du}{dp'} = \frac{du}{dw^c} \frac{dw^c}{dp'} = F' w^f - \bar{w} \frac{1}{1 + r}.$$

Note that the assumption that $F$ is developed and $H$ is developing naturally implies that $w^f > \bar{w}$. Since $F' > 0$,

$$\frac{du}{dp'} > 0.$$
In addition, noting that \( w^c \equiv \frac{1}{1+r} [w + p'(w^f - \overline{w})] \),

\[
\frac{du}{d(w^f - \overline{w})} = F' \frac{p'}{1+r} > 0 .
\] (2.8)

In summary, we have the following results. (1) The unemployment rate of university graduates in a developing country will increase as the probability of migration rises. (2) The unemployment rate of university graduates in a developing country will increase as the wage gap between the developed country and the developing country increases.

The benefit that education without migration confers is simply H’s mean wage rate of educated workers

\[
\overline{w} .
\]

When migration is a possibility, the expected payoff from the three stages described above is

\[
V \equiv pw^f + (1-p)\left\{ \int_{w^c}^{w^h} wdF(w) + F(w^c)\left[ p'w^f + (1-p)\overline{w} \right]/(1+r) \right\} 
= pw^f + (1-p)\left\{ \int_{w^c}^{w^h} wF'(w)dw + F(w^c)w^c \right\} .
\] (2.9)

We further assume that

\[
w^f > w^h .
\] (2.10)

To rule out the unreasonable possibility that all the educated are unemployed, we assume that

\[
w^c < w^h .
\] (2.11)

We next incorporate the cost of acquiring education. Our idea is that individuals differ in their abilities and familial background, hence in their cost of acquiring education. We normalize the size of (pre-migration) population of H to be Lebesgue measure 1. Suppose that an individual’s cost of obtaining education, \( c \), follows the following uniform distribution

\[
\tilde{c} \in [0, \Omega] .
\]

We assume that the (lifetime) income of an uneducated individual is constant, and we denote it by \( \Phi \). Then, recalling the assumption that only individuals with university degrees
have any chance of migrating, an individual will choose to acquire a university education if and only if

$$V - c \geq \Phi$$  

(2.12)

Let us define

$$c^* \equiv V - \Phi.$$  

(2.13)

It follows that an individual will obtain a university education if and only if his cost of education maintains

$$c \leq c^*.$$

Since $\tilde{c}$ follows a uniform distribution and the population size of the economy is of Lebesgue measure 1, both the proportion and the number of educated individuals are given by

$$\frac{c^*}{\Omega}.$$  

(2.14)

With these building blocks on site, we obtain the following lemma.

**Lemma 1:** There exists a positive level of $p$ at which the number of university graduates remaining in the developing country is higher than the number of university graduates in the developing country when $p = 0$, for any given $\alpha$, if $w^f > (3 + \alpha)\overline{w}$.

**Proof.** See Fan and Stark (2007).

Lemma 1 implies that a developing country may end up with more university graduates despite the brain drain of university graduates. Noting that there is a reduction in the population in the wake of migration, the lemma also implies that the developing country may end up with a higher fraction of educated individuals, despite the brain drain of university graduates.

**3. A short-run loss versus a long-run gain**

In this section we will show that in the short run, relaxation of migration, which leads to a brain drain and to “educated unemployment,” could result in a reduction of per-capita
output. Yet in the longer run (in the next generation), the legacy of a relaxed migration policy will prompt “take-off” of the economy. The latter result will be derived in a framework of rational expectations equilibrium.

Our analysis draws on the work of Azariadis and Drazen (1990), who emphasize the role of a “threshold externality” in economic development. They argue forcefully that the average level of human capital is a key factor for an economy’s “take-off”. Specifically, we assume that

$$\text{wage of the educated in the home country} = \begin{cases} \beta \bar{w} & \text{if } e \geq e^c \\ \frac{\bar{w}}{w} & \text{if } e < e^c \end{cases}$$

where $\beta > 1$, and $e$ denotes the proportion of the educated in the home country. Note that $e^c$ is the critical value that characterizes the “threshold externality” of average human capital. With labor being the only factor of production in the economy, an increase in the wage rate is tantamount to “take-off” of the economy. Since our modeling of the externality effect of human capital is different from the corresponding modeling in the related literature (cf. Mountford, and Stark and Wang), our model complements the received literature.

Since the number of individuals undertaking education is a function not only of the probability of migration, $p$, but also of the wage rate that awaits educated workers, we define

$$\xi = \begin{cases} \beta & \text{if } e \geq e^c \\ 1 & \text{if } e < e^c \end{cases}$$

We then note that $c^*$ is a function of $V$ and hence of $p$ and $\xi$, so we define

$$c^* \equiv c(p, \xi).$$ (3.1)

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6The assumption has been used widely in the literature (see, for example, Galor and Stark (1994) and Galor and Tsiddon (1996)). The “beneficial brain drain” literature has so far drawn on a single-period model or on a long-run steady state analysis, and hence is not suitable for the unraveling of the complete set of the dynamic costs and benefits, presumably tilting the analysis in favor of a more sympathetic view of the consequences of the migration of skilled workers.

7The concept and phenomenon of a “take-off” have been emphasized frequently in the development literature and are at the heart of many analyses by economic historians of the stages of economic growth (Rostow (1960)).

8The “big push” theory (for example, Murphy, Shleifer and Vishny (1989)) and the argument of a skill-induced technological change (for example, Acemoglu (1998)) both explain the endogenous determination of $\beta$. 
Then, under the migration prospect, the number of educated individuals \textit{remaining} in the home country is

\[
\frac{c(p, \xi)}{\Omega} - \left[ p \frac{c(p, \xi)}{\Omega} + (1 - p) p' \frac{c(p, \xi)}{\Omega} F(w^c) \right] = c(p, \xi)[(1 - p)(1 - p(1 + \alpha)F(w^c))] / \Omega. \tag{3.2}
\]

Note that the size of the population remaining in the home country, which we denote by \(n(p, \xi)\), decreases when \(p > 0\) in comparison with the case when \(p = 0\). Also, recall that to begin with, the size of the population of the economy is of Lebesgue measure 1. Then,

\[
n(p, \xi) = 1 - \left[ p \frac{c(p, \xi)}{\Omega} + (1 - p) p' F(w^c) \frac{c(p, \xi)}{\Omega} \right] = 1 - \frac{c(p, \xi)}{\Omega} + \frac{c(p, \xi)}{\Omega}(1 - p)[1 - p(1 + \alpha)F(w^c)]. \tag{3.3}
\]

From (3.2), we know that the fraction of the educated individuals out of the population remaining in the home country is

\[
e(p, \xi) \equiv c(p, \xi)(1 - p)[1 - p(1 + \alpha)F(w^c)] / n(p, \xi) \Omega. \tag{3.4}
\]

Then, “take-off” of \(H\) can be sustained (or achieved) by a rational expectations equilibrium if and only if

\[
e(p, \beta) \geq e^c. \tag{3.5}
\]

If (3.5) can be satisfied by a careful choice of \(p\), then “take-off” can occur in the current period. Yet even if (3.5) cannot be satisfied in the current period, it may be satisfied in the next period upon a careful choice of \(p\) in the current period, which increases the number of educated parents of the individuals in the next period.\(^9\)

In the following exposition we will use the subscript \(t\) to denote the current period, the subscript \(t - 1\) to denote the preceding period, and the subscript \(t + 1\) to denote the next period. When \(\xi\) takes the value 1, we will not write \(\xi\) explicitly unless the omission could cause confusion. (For example, to denote \(c(p, 1)\), we will write \(c(p)\).)

\(^9\)Since a larger \(\beta\) implies higher returns to education, we would expect \(e(p, \beta)\) to be an increasing function of \(\beta\). In addition, if \(e(0, \beta) < e^c\), a careful choice of \(p (> 0)\) can reverse this inequality.
Resorting to an assumption which appears to have gained wide adherence - that the cost of acquiring education decreases with parental human capital (that is, the number of parents who have acquired a university education), we write

\[
\frac{d\Omega_{t+1}}{dc_t} < 0. \tag{3.6}
\]

The importance of parental human capital for an individual’s educational attainment has been consistently confirmed in the empirical literature. (For a helpful survey see Hanushek (1996).)

We are now in a position to state and prove the following proposition.

**Proposition 1:** (1) If (3.5) cannot be satisfied so that “take-off” does not occur in the current period, the prospect of migration entails a decline in the economy’s per-capita output in the short run. (2) However, a careful choice of \( p \) in both the current period and the next period can facilitate “take-off” of the economy in the next period.

**Proof.** (1) If “take-off” does not occur in the current period, the prospect of migration will result in a loss of average (per-capita) output.

To facilitate a comparison between the case in which \( p > 0 \) and the case in which \( p = 0 \), we divide the individuals into three distinct categories (for the case in which \( p > 0 \)):

(i) Individuals who do not acquire education;

(ii) Individuals who acquire education and fail to secure work abroad;

(iii) Individuals who acquire education and migrate.

(i) Individuals of the first type do not acquire education when \( p > 0 \). From the analysis in the previous section we know that they would not have acquired education when \( p = 0 \). Thus, the prospect of migration has no impact on their (net) earnings which, in either case, are equal to the wage of the uneducated, \( \Phi \).

(ii) As to individuals of the second type, the prospect of migration results in some of them receiving lower net earnings than the earnings that they would have received when
$p = 0$. This comes about through two channels: (a) The prospect of migration prompts “too many” individuals to acquire education; (b) the prospect of migration causes “educated unemployment”.

(a) When there is no prospect of migration, the number of educated individuals is $\frac{c(0)}{\Omega}$. When $p > 0$, the number of educated individuals is $\frac{c(p)}{\Omega}$. The number of educated individuals who would not choose to be skilled without the prospect of migration is then

$$\frac{c(p)}{\Omega} - \frac{c(0)}{\Omega}.$$ 

Note that the proportion of these individuals who do not migrate is

$$(1 - p)[1 - p'F(w^c)] = (1 - p)[1 - p(1 + \alpha)F(w^c)].$$

Thus, in the presence of a prospect of migration, the number of educated individuals remaining in the home country who have acquired a higher education “wrongly” is not less than

$$\left[\frac{c(p)}{\Omega} - \frac{c(0)}{\Omega}\right](1 - p)[1 - p(1 + \alpha)F(w^c)].$$

For these individuals, the cost of their education is in the domain $[c(0), c(p)]$, and the distribution of that cost in this domain is uniform. Thus, the average cost of education for these individuals is

$$\frac{c(0) + c(p)}{2}. \quad (3.8)$$

The (expected) benefit of education (in comparison with no education) for any individual who remains in the home country is less than\(^\text{10}\) or equal to $\pi - \Phi$. (Since the number of individuals is a continuum, the expected value is equal to the average value.)

When $p = 0$, $V = \bar{w}$. Hence, from (2.13) and the definition $c^* = c(p)$

$$c(0) = V - \Phi = \bar{w} - \Phi. \quad (3.9)$$

Thus, the average net loss per individual is not less than

$$\frac{c(0) + c(p)}{2} - (\bar{w} - \Phi) > c(0) - (\bar{w} - \Phi) = 0. \quad (3.10)$$

\(^\text{10}\)It can be less because some individuals may choose to become unemployed, yet the unemployment is ex post inefficient if they fail to go abroad.
(b) From Assumption (2.4), no educated individual will choose to be unemployed in the absence of a prospect of migration. Therefore, the (discounted) income of some of the educated individuals remaining in the home country would have been higher had they not chosen to be unemployed (in the sense of an *ex post* consideration). From Figure 1, we can see that the total number of unemployed educated individuals before the second lottery of migration occurs (i.e. in Stage 2) is $F(w^c) \frac{c(p)}{\Omega}(1 - p)$. Therefore, the number of these unemployed educated individuals who remain in the home country is

$$F(w^c) \frac{c(p)}{\Omega}(1 - p)[1 - p(1 + \alpha)] . \quad (3.11)$$

If these individuals had worked rather than been unemployed, their average income would have been

$$E(w|w^l \leq w \leq w^c) = \frac{\int_{w^l}^{w^c} wdF(w)}{F(w^c)} , \quad (3.12)$$

where $E$ is the expectation operator.

However, because they chose to wait, their expected earnings are

$$\bar{w} \over 1 + r . \quad (3.13)$$

(Again, note that the number of individuals is a continuum, hence the expected value is equal to the average value.)

Thus, recalling (2.4), we know that the average loss per individual is

$$E(w|w^l \leq w \leq w^c) - \frac{\bar{w}}{1 + r} > w^l - \frac{\bar{w}}{1 + r} > 0 , \quad (3.14)$$

where the inequality sign in (3.14) arises from (2.4).

The preceding discussion shows that for the set of individuals who remain in the home country when $p > 0$, that is, individuals of types (i) and (ii), some receive lower net earnings than when $p = 0$, while others receive the same net earnings. Thus, the average earnings of type (i) and type (ii) individuals when $p > 0$ are lower than when $p = 0$. We next show that the departure of educated individuals further reduces the average income.
(iii) Had \( p = 0 \), the individuals who would have acquired an education as a fraction of the individuals who would have acquired education had \( p > 0 \) is
\[
\frac{c(0)/\Omega}{c(p)/\Omega} = \frac{c(0)}{c(p)}
\]

When \( p = 0 \), the average income of type (iii) individuals who would have acquired education, net of the education cost, would have been \( \bar{w} - \frac{c(0)}{2} \). Recall that the earnings of the uneducated are \( \Phi \). Thus, when \( p = 0 \), the average income of individuals of type (iii) is
\[
\frac{c(0)}{c(p)}[\bar{w} - \frac{c(0)}{2}] + \frac{1 - c(0)}{c(p)}\Phi.
\]
(3.15)

When \( p = 0 \), the average income of all individuals is
\[
\frac{c(0)}{\Omega}[\bar{w} - \frac{c(0)}{2}] + \frac{1 - c(0)}{\Omega}\Phi.
\]
(3.16)

Because \( \Omega > c(p) \), and \( \bar{w} - \frac{c(0)}{2} > \bar{w} - c(0) = \Phi \) (recall (2.4)), we have that
\[
\frac{c(0)}{c(p)}[\bar{w} - \frac{c(0)}{2}] + \frac{1 - c(0)}{c(p)}\Phi > \frac{c(0)}{\Omega}[\bar{w} - \frac{c(0)}{2}] + \frac{1 - c(0)}{\Omega}\Phi.
\]

Thus, the average income of the individuals whom the home country loses through migration would have been higher than the national average when \( p = 0 \). Thus, when \( p = 0 \), the average income of individuals of type (i) and type (ii) is lower than the average income of individuals of type (i), type (ii), and type (iii). Therefore, the loss of educated individuals through migration further reduces the average income in the economy.

(2) Note that from (3.4),
\[
\frac{de_t(p_t, \xi)}{d\Omega_t} < 0
\]
(3.17)
for any given \( p_t \) and \( \xi \). Since, recalling (3.6),
\[
\frac{d\Omega_t}{de_{t-1}} < 0,
\]
it follows that
\[
\frac{de_t(p_t, \xi)}{de_{t-1}} > 0.
\]
(3.18)
Thus, when $p_{t-1}$ is chosen in such a way that $p_{t-1} = p^o > 0$ and $e(p^o, 1) > e(0, 1)$, noting (3.18), we have

$$e_t(p_t^*, \beta)|_{p_{t-1}=p^o} \geq e_t(p_t^{**}, \beta)|_{p_{t-1}=p^o} > e_t(p_t^{**}, \beta)|_{p_{t-1}=0}$$ (3.19)

where the notation "$e_t(p_t^*, \beta)|_{p_{t-1}=p^o}$" means the fraction of the population remaining in the home country who are educated when $p_{t-1} = p^o$ and $p_t = p_t^*$, and where

$$p_t^* = \arg \max e_t(p_t, \beta)|_{p_{t-1}=p^o}$$

and

$$p_t^{**} = \arg \max e_t(p_t, \beta)|_{p_{t-1}=0}.$$ 

Hence, when $e^c$ is in the region

$$e_t(p_t^*, \beta)|_{p_{t-1}=p^o} > e^c > e_t(p_t^{**}, \beta)|_{p_{t-1}=0}$$ (3.20)

"take-off" is possible in period $t$ in a framework of rational expectations equilibrium only if migration was allowed in the preceding period so that more parents chose to become educated.

Proposition 1 analyzes the negative and the positive welfare implications of migration by skilled individuals in a unified framework. In the short run, we encounter three possible negative consequences: migration leads to a reduction in the “stock” of better-educated individuals, which in turn reduces average income; when a fraction of the educated individuals who otherwise would have worked are lured to form human capital only to end up unemployed, output shrinks; since the possibility of migration motivates individuals to acquire higher education, when some of them remain in the home country, the returns to their education will turn out to fall short of the costs of their education. Unless the economy “takes off,” these individuals’ overeducation is socially inefficient in the short run.

However, we next demonstrate that in the long run (one generation down the road), the legacy of a relaxed migration policy prompts “take-off” of the economy. Our results are derived in a framework of rational expectations equilibrium: the brain drain is accompanied
by a “brain gain;” the ensuing “brain gain” can result in a higher average level of human
capital in the home country; the higher average level of human capital can prompt “take-off”
of the economy. In such a setting, overeducation can become dynamically efficient (due to
the intergenerational externality effect of human capital) even though it may be statically
inefficient. Thus, Proposition 1 implies that a relaxation in migration policy in both periods
is conducive to achieving the benefit of long-run growth.

4. The prospect of a welfare gain

In this section we examine the welfare implications of “take-off” in the next period. We
use $L^s$ to denote the short-run loss in terms of average income arising from the prospect
of migration, and $G^l$ to denote the benefit measured in terms of the average income in the
next period arising from the prospect of migration less the average income that would have
obtained in the absence of such a prospect. We thus define the social welfare function as
follows:

$$-L^s + \rho(G^l)$$

(4.1)

where $\rho$ is the social discount rate across generations. Then, we have the following propo-
sition.

**Proposition 2:** Suppose that the economy takes off in the next period if and only if migration
is allowed. If $\beta$ is sufficiently large such that

$$\beta > \frac{0.5[(1 - p)(c(p))^2 - (c(0))^2] + \Phi(c(p) - c(0))}{\rho c(0)\bar{w}} + \frac{r + \rho + \rho \rho}{\rho (1 + r)}$$

(4.2)

migration of educated individuals will confer a welfare gain to the individuals remaining in
the home country despite the phenomena of brain drain and “educated unemployment”.

**Proof.** See Appendix A.

Proposition 2 implies that in spite of the incorporation of an additional cost of migration
for a developing country, the insight that the brain drain can confer a benefit to the country
is still retained. Rather than causing human capital drain and output contraction, the
migration of educated workers entails human capital gain and output growth. An analysis of the entire dynamics associated with the response of educated workers to the prospect of migration therefore raises the intriguing possibility that the devil is, in fact, an angel. The results are more powerful than those reported early on since the prospect of migration is taken to entail both depletion of human capital and unemployment of human capital, which renders it more difficult to hold migration as a catalyst for growth.

In addition, when “take-off” occurs, we have the following proposition.

**Proposition 3:** After “take-off,” the unemployment rate of the educated is lower than that prior to “take-off”.

**Proof.** Prior to “take-off”, we know, following (2.3) and (2.5), that the unemployment rate of the educated is

\[ u^b \equiv F(w^c) = F\left[\frac{(1 - p')\bar{w}}{1 + r} + \frac{p'w_f}{1 + r}\right]. \tag{4.3} \]

After “take-off,” the fraction of the educated who are unemployed is

\[ u^a \equiv P(\beta \hat{w} \leq w^{cc}) = F\left(\frac{w^{cc}}{\beta}\right) \tag{4.4} \]

where \( w^{cc} \) is the equivalent of \( w^c \) in (2.3), that is,

\[ w^{cc} = \frac{1}{1 + r}[(1 - p')\beta \bar{w} + p'w_f]. \tag{4.5} \]

Thus,

\[
\begin{align*}
  u^a &= F\left(\frac{w^{cc}}{\beta}\right) \\
  &= F\left(\frac{(1 - p')\beta \bar{w} + p'w_f}{(1 + r)\beta}\right) \\
  &= F\left[\frac{(1 - p')\bar{w}}{1 + r} + \frac{p'w_f}{1 + r} \right]. \tag{4.6}
\end{align*}
\]

Comparing (4.3) and (4.6) and noting that \( \beta > 1 \) and \( F' > 0 \), we have

\[ u^b > u^a. \tag{4.7} \]

\[ \blacksquare \]
Proposition 3 states that “take-off” bites into the unemployment rate. The intuition is straightforward. After “take-off,” the domestic wage rate of educated workers increases. Hence, the relative benefit of waiting for overseas employment decreases. This reduces the unemployment rate.

5. Simulation

We conduct simulation exercises aimed at fleshing out the channels that were identified in the analysis undertaken in the preceding sections. We divide this section into 5 subsections. Subsection 5.1 specifies the parameters. In relation to the proof of Proposition 1, subsection 5.2 analyzes the cost of “educated unemployment”; subsection 5.3 examines the cost of overeducation; subsection 5.4 discusses the direct cost of a brain drain; subsection 5.5 investigates the brain gain.

5.1. Parameter specifications

We specify the parameters as follows:

$$\alpha = 0, \ w^l = 1, \ w^h = 2, \ w^f = 5, \ r = 0.5, \ \Phi = 1.2 \quad (5.1)$$

$\bar{w}$ follows a uniform distribution over the domain $[1, 2]$. Therefore we get

$$\bar{w} = 1.5 \quad (5.2)$$

This implies that the wage rate in F is 3.3 ($\frac{5}{1.5}$) times the average wage rate for the skilled in H. Also, note that it is possible that $\Phi > w^l$ since schooling involves an opportunity cost of not working. Moreover, we specify that the (initial) value of $\Omega$, the upper bound of the cost of acquiring education, is 3.

From (5.1) and (5.2), and recalling (2.3), we get

$$w^c = \frac{1}{1 + r}[1 - (p^l)\bar{w} + p^l w^f] = 1 + \frac{7}{3}p \quad (5.3)$$

Since $w^c < w^h$, we assume that

$$1 + \frac{7}{3}p < 2.$$
namely that
\[ p < \frac{3}{7} \]  
(5.4)

From (5.3), we get
\[ F(w^c) = \int_1^{1 + \frac{7}{3} p} dw = \frac{7}{3}p . \]  
(5.5)

Inserting (5.1), (5.3), and (5.5) into (2.9), we get
\[ V = pwf + (1 - p)\left[ \int_{w^c}^{wh} wF'(w)dw + F(w^c)w^c \right] \]
\[ = 1.5 + 3.5p + \frac{49}{18}p^2 - \frac{49}{18}p^3 . \]  
(5.6)

Then, from (2.13), we have
\[ c(p) = V - \Phi = 0.3 + 3.5p + \frac{49}{18}p^2 - \frac{49}{18}p^3 . \]

Then, recalling (3.2), when there is a prospect of migration, the number of the educated individuals, say university graduates, remaining in the developing country is
\[ R(p) \equiv \frac{c(p)}{\Omega} - \left[ p\frac{c(p)}{\Omega} + (1 - p)p'\frac{c(p)}{\Omega}F(w^c) \right] \]
\[ = (0.3 + 3.5p + \frac{49}{18}p^2 - \frac{49}{18}p^3)[(1 - p)(1 - \frac{7}{3}p^2)]/\Omega . \]  
(5.7)

Since the number of uneducated individuals (who do not migrate) is \(1 - \frac{c(p)}{\Omega}\), the total number of individuals remaining in \(H\) is
\[ R(p) + 1 - \frac{c(p)}{\Omega} . \]  
(5.8)

5.2. The cost of “educated unemployment”

Inserting (5.1), (5.3), (5.5), and \(\Omega = 3\) into (3.11), we get that the number of the unemployed educated individuals who remain in the home country is
\[ U(p) \equiv (1 - p)F(w^c)\frac{c(p)}{\Omega}(1 - p) \]
\[ = \frac{7}{9}p(1 - p)^2(0.3 + 3.5p + \frac{49}{18}p^2 - \frac{49}{18}p^3) . \]  
(5.9)
The proportion of these individuals as a percentage of the total number of individuals who remain in the home country is

\[ u(p) = \frac{U(p)}{R(p) + 1 - \frac{c(p)}{\Omega}}. \tag{5.10} \]

Also, as discussed in Section 2, a simple indicator of the unemployment rate among the educated individuals is \( F(w^c) \).

From (3.14), we know that the average loss for these individuals is

\[ E(w^l \leq w \leq w^c) - \frac{\overline{w}}{1 + r} = \frac{1 + 1 + \frac{7}{3}p}{2} - \frac{1.5}{1 + 0.5} = \frac{7}{6}p. \tag{5.11} \]

This earning loss in terms of the percentage of these individuals’ average earnings in the absence of unemployment is then

\[ l^u = \frac{\frac{7}{6}p}{E(w^l \leq w \leq w^c)} = \frac{\frac{7}{6}p}{1 + \frac{7}{6}p} = \frac{7p}{6 + 7p}. \tag{5.12} \]

Then, we have the following Table:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( l^u ) (%) (average cost of “educated unemployment”)</th>
<th>( F(w^c) ) (%) (unemployment rate of the educated)</th>
<th>( u ) (%) (total unemployment rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>1.15</td>
<td>2.33</td>
<td>.26</td>
</tr>
<tr>
<td>2%</td>
<td>2.28</td>
<td>4.67</td>
<td>.56</td>
</tr>
<tr>
<td>3%</td>
<td>3.38</td>
<td>7.00</td>
<td>.90</td>
</tr>
<tr>
<td>4%</td>
<td>4.46</td>
<td>9.33</td>
<td>1.28</td>
</tr>
<tr>
<td>5%</td>
<td>5.51</td>
<td>11.67</td>
<td>1.71</td>
</tr>
<tr>
<td>6%</td>
<td>6.54</td>
<td>14.00</td>
<td>2.17</td>
</tr>
<tr>
<td>7%</td>
<td>7.55</td>
<td>16.33</td>
<td>2.66</td>
</tr>
<tr>
<td>8%</td>
<td>8.54</td>
<td>18.67</td>
<td>3.20</td>
</tr>
<tr>
<td>9%</td>
<td>9.50</td>
<td>21.00</td>
<td>3.77</td>
</tr>
<tr>
<td>10%</td>
<td>10.45</td>
<td>23.33</td>
<td>4.37</td>
</tr>
</tbody>
</table>

From Table 1 we see that as the probability of migration increases, both the unemployment rate of the educated and the average loss for these unemployed individuals increases. For example, if \( p = 10\% \), then the unemployment rates among the educated and among the entire population are, respectively, 23.33\% and 4.37\%, and the average (percentage) loss for these unemployed individuals is 10.45\%.
5.3. The cost of overeducation

From (3.7) and (5.7), we know that when there is a prospect of migration, the number of educated individuals remaining in the home country who have acquired a higher education “wrongly” is not less than

\[ O(p) \equiv R(p) - R(0) . \]  \hspace{1cm} (5.13)

The proportion of these individuals as a percentage of the educated individuals who remain in the home country is

\[ \lambda(p) \equiv \frac{O(p)}{R(p)} ; \]  \hspace{1cm} (5.14)

the proportion of these individuals as a percentage of the total number of individuals who remain in the home country is

\[ \eta(p) \equiv \frac{O(p)}{R(p) + 1 - c(p)/\Omega} . \]  \hspace{1cm} (5.15)

From (3.10), we know that for these individuals, the average net loss per individual is not less than

\[ \frac{c(0) + c(p)}{2} - (\bar{w} - \Phi) . \]  \hspace{1cm} (5.16)

If an individual does not acquire an education, his net earning are \( \Phi \). Thus, the percentage loss is not less than

\[ l^p \equiv \frac{\frac{c(0)+c(p)}{2} - (\bar{w} - \Phi)}{\Phi} = \frac{c(0) + c(p) - 0.6}{2.4} . \]  \hspace{1cm} (5.17)

Then, we have the following Table:
From Table 2 we see that as the probability of migration increases, the proportion of overeducated individuals increases no matter whether the proportion is measured as a percentage of the educated individuals who remain in the home country or as a percentage of the total number of individuals who remain in the home country. For example, if $p = 10\%$, the proportion of overeducated individuals is close to $50\%$ of the educated individuals who remain in the home country, and about $10\%$ of the entire population. Also, as the probability of migration increases, the average net loss of overeducated individuals increases. For example, if $p = 10\%$, then the average loss of overeducation is $15.60\%$.

### 5.4. The direct cost of a brain drain

Suppose that a certain number of educated individuals migrate from H to F, which results in a brain drain. The direct cost of the brain drain is measured as the difference between the average income when no migration is allowed and the (maximal) average income when a certain number of educated individuals migrate. Specifically, we proceed in two steps. (i) We calculate the average net income when migration is not allowed. We denote this average income by $Z^1$. Clearly,

$$Z^1 = \frac{c(0)}{\Omega} \left[ \bar{\pi} - \frac{c(0)}{2} \right] + \left[ 1 - \frac{c(0)}{\Omega} \right] \Phi.$$  \hfill (5.18)
(ii) From the preceding analysis we know that the number of individuals who migrate is

\[
M(p) = \frac{c(p)}{\Omega} p + p'(1 - p)\frac{c(p)}{\Omega} F(w^c) \\
= (0.3 + 3.5p + \frac{49}{18}p^2 - \frac{49}{18}p^3)[p + (1 - p)\frac{7}{3}p^2]/\Omega.
\]

(5.19)

For the remaining individuals, if they could anticipate that they would stay in H, then neither “educated unemployment” nor overeducation would occur. In this hypothetical scenario, we calculate the average income for the remaining individuals (educated and uneducated) in H, which we denote by \( Z^2 \). Note that this calculating procedure eliminates the influence of “educated unemployment” and overeducation, which allows us to calculate the direct cost of the brain drain. Then, the direct loss from the brain drain, which is denoted by \( D \), is

\[
D = Z^1 - Z^2.
\]

To calculate \( Z^2 \), we first calculate the average income of those individuals who migrate (in the hypothetical scenario that they anticipate \( p = 0 \)), which is

\[
I(p) \equiv \frac{c(0)}{c(p)}[\bar{w} - \frac{c(0)}{2}] + [1 - \frac{c(0)}{c(p)}]\Phi.
\]

(5.20)

Recall that the total number of individuals in H in the absence of migration is one. Then, we know that the total income of the remaining individuals (in this hypothetical scenario) is

\[
1Z^1 - M(p)I(p) = Z^1 - M(p)I(p).
\]

(5.21)

Thus,

\[
Z^2 = \frac{Z^1 - M(p)I(p)}{1 - M(p)}.
\]

(5.22)

Then, the direct loss from the brain drain in percentage terms is

\[
d \equiv \frac{Z^1 - Z^2}{Z^1}.
\]

(5.23)

Then, we have the following Table:
Table 3

<table>
<thead>
<tr>
<th>p</th>
<th>$Z_1^p$</th>
<th>$M(p)$ (% of total population)</th>
<th>$Z_2$</th>
<th>$d(%)$ (average direct cost of migration)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>1.215</td>
<td>1.11</td>
<td>1.2149</td>
<td>.011</td>
</tr>
<tr>
<td>2%</td>
<td>1.215</td>
<td>.26</td>
<td>1.2147</td>
<td>.023</td>
</tr>
<tr>
<td>3%</td>
<td>1.215</td>
<td>.44</td>
<td>1.2146</td>
<td>.034</td>
</tr>
<tr>
<td>4%</td>
<td>1.215</td>
<td>.65</td>
<td>1.2144</td>
<td>.046</td>
</tr>
<tr>
<td>5%</td>
<td>1.215</td>
<td>.89</td>
<td>1.2143</td>
<td>.058</td>
</tr>
<tr>
<td>6%</td>
<td>1.215</td>
<td>1.18</td>
<td>1.2141</td>
<td>.070</td>
</tr>
<tr>
<td>7%</td>
<td>1.215</td>
<td>1.50</td>
<td>1.2140</td>
<td>.082</td>
</tr>
<tr>
<td>8%</td>
<td>1.215</td>
<td>1.86</td>
<td>1.2139</td>
<td>.094</td>
</tr>
<tr>
<td>9%</td>
<td>1.215</td>
<td>2.27</td>
<td>1.2137</td>
<td>.107</td>
</tr>
<tr>
<td>10%</td>
<td>1.215</td>
<td>2.72</td>
<td>1.2136</td>
<td>.119</td>
</tr>
</tbody>
</table>

From Table 3 we see that $d$ increases with the probability of migration. However, $d$ is only 0.119% even when $p = 10\%$, which implies that the direct cost of migration is quite small. The intuition is that most of the population in a developing country is uneducated, and it is this uneducated population that mainly determines the average income in the country. Thus, even if a significant proportion of the educated migrate, the impact on the average income of the developing country can be relatively small.

In summary, the simulation conducted thus far shows that the cost of the two new negative consequences of migration introduced in this paper, namely “educated unemployment” and overeducation, can amount to significant losses for the individuals affected who may constitute a substantial proportion of the educated individuals. In addition, in per capita terms, the cost of a brain drain may be relatively small if the proportion of educated individuals in the economy is small.

5.5. The brain gain

From (5.7) and (5.8), the proportion of university graduates remaining in the developing country as a percentage of the total number of individuals who remain in the country is

$$k(p) \equiv \frac{R(p)}{R(p) + 1 - c(p)/\Omega}.$$  \hspace{1cm} (5.24)

Then, we have the following Table:
From Table 4 we see that as \( p \) increases from 0 to 10%, the proportion of university graduates remaining in the developing country more than doubles.

Suppose that 20% of educated workers is the threshold level above which “take-off” will occur. From the preceding calculations we see that only \( p = 10\% \) can satisfy this condition. If the home country’s government cannot set \( p = 10\% \) (for example, due to a constraint on migration set by the foreign government), then “take-off” cannot occur in the current period.

Now we specify

\[
\Omega_t = \Omega_{t-1} - 15(k_{t-1} - 0.1) \tag{5.25}
\]

where \( k_t \) is the value of \( k(p) \) at time \( t \). As in Section 3, we refer time \( t - 1 \) as in the preceding period and time \( t \) as in the current period.

Also, for simplicity, we assume that the government of the home country can only set \( p = 5\% \) in the current period. Then, noting Table 4 and from (5.24) and (5.25), we have the following Table:
Recall the assumption that 20% of educated workers is the threshold level above which “take-off” will occur. From Table 5 we see that “take-off” will occur in the current period if and only if \( k_{t-1} \) (that is, \( k(p) \times 100 \) in the preceding period) is greater or equal to 15.29, that is, correspondingly, the home country’s government sets \( p_{t-1} \geq 5\% \) in the preceding period. Thus, the simulation shows that a relaxation in migration policy in both the current period and the preceding period can facilitate “take-off” in a developing economy in the current period.

6. Conclusions

Extending both the “harmful brain drain” literature and the “beneficial brain gain” literature, this paper analyzes both the negative and the positive impact of migration by skilled individuals in a unified framework. The paper extends the received literature on the “harmful brain drain” by showing that in the short run, international migration can result in “educated unemployment” and in overeducation in developing countries, as well as in a brain drain from these countries. Adopting a dynamic framework, it is then shown that due to the positive externality of the prevailing, economy-wide endowment of human capital on the formation of human capital, a relaxation in migration policy in both the current and the preceding period can facilitate “take-off” of a developing country in the current period.
Thus, it is suggested that while controlled migration by skilled individuals may reduce the social welfare of those who stay behind in the short run, it improves it in the long run.

The reason we resort to the “educated unemployed” assumption is that we seek to track the implications of the removal of workers from gainful employment in their home country, a separation that occurs because they have the prospect of employment abroad. Our argument does not hinge then on workers being unemployed at home; if workers who failed to secure employment abroad while remaining at home were then to migrate and seek employment while living abroad, and then, if unsuccessful, were to return and take up work at home, the result would be the same - as long as seeking employment while abroad did not make it more likely to succeed. It is the removal of workers from employment, not their location when seeking work abroad, that matters.

At the heart of our analysis is the idea that allowing some individuals to work abroad implies not only a brain drain and “educated unemployment” at home, but also, because the prospect of migration raises the expected returns to higher education, a “brain gain;” the developing country ends up with a higher proportion of educated individuals. Indeed, the brain drain is a catalyst for a “brain gain”. More importantly, due to the positive externality of the prevailing, economy-wide endowment of human capital on the formation of human capital, a relaxation in migration policy in both the current period and the preceding period can facilitate “take-off” in a developing economy in the current period. Thus, our analysis points to a new policy tool that could yield an improvement in the well-being of the population of a developing economy: a controlled migration of educated workers. Somewhat counterintuitively, it is the departure of human capital that sets in motion a process of acquisition of human capital which, in turn, may well lead to economic betterment for all.

We conduct our analysis in the framework of partial equilibrium, assuming away a detailed analysis of the determination of the wage rate. Although an analysis based on a general-equilibrium framework will constitute a useful extension, we contend that such an extension will not change the qualitative results of our analysis. For example, if we are
interested in exploring significant economy-wide repercussions of migration, would not it be appropriate for us to assume, as in basic textbook reasoning, that the departure of workers raises wages at home and lowers wages abroad? Not really. The essence of our argument is that a small probability of working abroad could trigger large repercussions such as the ones that we allude to. As such, the small migration that takes place need not be accompanied by any discernible changes in wage rates either at home or abroad. Moreover, there are two main repercussions to the formation of human capital that tend to impact in opposite directions, hence could cancel each other out: on the one hand, an increased supply of human capital at home could lower the returns to human capital at home; on the other hand, the increased supply of human capital could confer positive externalities, hence raise the returns to human capital. The partial equilibrium setting could then be akin to that which would be yielded by a general equilibrium configuration.

Two additional comments in support of the robustness of our analysis are in order. First, it might be argued that if a fraction of the educated workers exit the home economy and if educated workers withdraw their labor from the home economy, then the wage paid to educated workers could be expected to rise. We have, however, already pointed out that such a wage change may not occur since these two responses coincide with the remainder of the educated workforce being more educated, an occurrence that is in direct response to the prospect of migration. Furthermore if, contrary to our assumption, the said two responses will indeed entail an increase in the home-economy’s wage for the educated, then the effect of the enhancement of expected earnings through the prospect of migration will only be amplified by the higher home-country wage, and our claim that prospect of migration impacts favorably on human capital formation will become even more compelling. Second, the assumption of a homogenous workforce eases our exposition. In related work, Stark and Wang (2002) have shown that, albeit at the cost of some mathematical complexity, incorporating the alternative assumption of a heterogeneous workforce yields results akin to those derived upon assuming a homogeneous workforce.
Appendix A: Proof of Proposition 2

From the proof of Proposition 1, recalling (3.16), we know that when \( p = 0 \), the average income of all the individuals is

\[
\frac{c(0)}{\Omega} \left[ \overline{w} - \frac{c(0)}{2} \right] + \left[ 1 - \frac{c(0)}{\Omega} \right] \Phi .
\]

When \( p > 0 \), and recalling (3.2), the number of educated individuals remaining in the home country is,

\[
(1 - p)[1 - p(1 + \alpha)F(w^c)]\frac{c(p)}{\Omega} ,
\]

and the number of uneducated individuals is \( 1 - \frac{c(p)}{\overline{w}} \). Therefore, for all the individuals remaining in the home country, if no one had chosen to be unemployed, the total (net) income would have been

\[
(1 - p)[1 - p(1 + \alpha)F(w^c)]\frac{c(p)}{\Omega} \left[ \overline{w} - \frac{c(p)}{2} \right] + \left[ 1 - \frac{c(p)}{\Omega} \right] \Phi . \tag{6.1}
\]

Furthermore, from (3.11), we know that the number of the individuals who become unemployed and remain in the home country is

\[
F(w^c)c(p)(1 - p)[1 - p(1 + \alpha)]/\Omega .
\]

The average income of these individuals, had they chosen to work rather than become unemployed, would have been \( E(w|w^l \leq w \leq w^c) \), whereas, recalling (3.13), their average income when choosing unemployment is \( \frac{\overline{w}}{1 + r} \). Since, recalling (3.14), the average net cost is

\[
E(w|w^l \leq w \leq w^c) - \frac{\overline{w}}{1 + r} ,
\]

the total cost is

\[
[E(w|w^l \leq w \leq w^c) - \frac{\overline{w}}{1 + r}]F(w^c)c(p)(1 - p)[1 - p(1 + \alpha)]/\Omega . \tag{6.2}
\]

Thus, when \( p > 0 \), for all the individuals remaining in the home country, their total income (income if all were employed less the income lost due to unemployment) is equal to

\[
\Lambda \equiv (1 - p)[1 - p(1 + \alpha)F(w^c)]\frac{c(p)}{\Omega} \left[ \overline{w} - \frac{c(p)}{2} \right] + \left[ 1 - \frac{c(p)}{\Omega} \right] \Phi - \left[ E(w|w^l \leq w \leq w^c) - \frac{\overline{w}}{1 + r} \right]F(w^c)c(p)(1 - p)[1 - p(1 + \alpha)]/\Omega . \tag{6.3}
\]

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For expositional simplicity, we define \( \Gamma \equiv (1-p)[1-p(1+\alpha)F(w^c)] \). Since \( \Gamma > (1-p)[1-p(1+\alpha)] \), total income is

\[
\Lambda > \Gamma \frac{c(p)}{\Omega} \left[ \frac{\bar{w} - c(p)}{2} \right] + [1 - \frac{c(p)}{\Omega}] \Phi - \frac{\bar{w}}{1 + r} F(w^c) c(p) \Gamma / \Omega \\
\geq \Gamma \frac{c(p)}{\Omega} \left[ \frac{\bar{w} - c(p)}{2} \right] + [1 - \frac{c(p)}{\Omega}] \Phi - \frac{\bar{w}}{1 + r} F(w^c) c(p) \Gamma / \Omega \\
= \frac{\bar{w}}{1 + r} \Gamma \frac{c(p)}{\Omega} - \Gamma \frac{(c(p))^2}{2 \Omega} + [1 - \frac{c(p)}{\Omega}] \Phi .
\]

(6.4)

Assuming that the condition, \( w^f > (3 + \alpha) \bar{w} \), is satisfied, we know from Lemma 1 that

\[
\Gamma \frac{c(p)}{\Omega} = (1-p)[1-p(1+\alpha)F(w^c)] \frac{c(p)}{\Omega} > \frac{c(0)}{\Omega} .
\]

Therefore, we have

\[
\Lambda > \frac{c(0)}{\Omega} \frac{\bar{w}}{1 + r} - \Gamma \frac{(c(p))^2}{2 \Omega} + [1 - \frac{c(p)}{\Omega}] \Phi .
\]

Note that since the entire population is normalized to be one, the total number of the individuals remaining in the home country is less than one. Noting that \( \Gamma < 1 - p \), we know that the average net income of these individuals is greater than

\[
\frac{c(0)}{\Omega} \frac{\bar{w}}{1 + r} - \Gamma \frac{(c(p))^2}{2 \Omega} + [1 - \frac{c(p)}{\Omega}] \Phi > \frac{c(0)}{\Omega} \frac{\bar{w}}{1 + r} - (1-p) \frac{(c(p))^2}{2 \Omega} + [1 - \frac{c(p)}{\Omega}] \Phi
\]

(6.5)

When \( p = 0 \), recalling (3.16), the average income of all individuals is

\[
\frac{c(0)}{\Omega} \left[ \frac{\bar{w} - c(0)}{2} \right] + [1 - \frac{c(0)}{\Omega}] \Phi .
\]

Thus, the short-run loss in terms of average income arising from the migration prospect is less than

\[
L = \frac{c(0)}{\Omega} \left[ \frac{\bar{w} - c(0)}{2} \right] + [1 - \frac{c(0)}{\Omega}] \Phi - \frac{c(0)}{\Omega} \frac{\bar{w}}{1 + r} + (1-p) \frac{(c(p))^2}{2 \Omega} \\
- [1 - \frac{c(p)}{\Omega}] \Phi \\
= \frac{rc(0)}{(1 + r) \Omega} \bar{w} + \frac{(1-p)(c(p))^2 - (c(0))^2}{2 \Omega} + \Phi \frac{(c(p) - c(0))}{\Omega} .
\]

(6.6)
Consider now the gain to the home country if “take-off" occurs in the next period. We first note that a feasible scenario is for the government to set \( p = 0 \) after the economy takes off. Hence, the maximal average income of the economy when \( p \) is optimally chosen is not less than that when \( p = 0 \). Next, we note that when \( p = 0 \) and after the economy takes off, the number of individuals who choose to be skilled is greater than the corresponding number before the economy takes off, \( \frac{c(0)}{\Omega} \). This increase in the number of educated individuals will increase average national income, since the gain from receiving education is greater than the cost of acquiring education. Thus, after the economy takes off, average income in the economy is greater than that when \( p = 0 \) and \( \frac{c(0)}{\Omega} \) fraction of individuals receive higher education, namely

\[
\frac{c(0)}{\Omega} [\beta \overline{w} - \frac{c(0)}{2}] + \left[ 1 - \frac{c(0)}{\Omega} \right] \Phi.
\]

Then, the benefit measured in terms of the average income in the next period arising from the prospect of migration less the average income that would have obtained in the absence of such a prospect is greater than

\[
G \equiv \frac{c(0)}{\Omega} [\beta \overline{w} - \frac{c(0)}{2}] + \left[ 1 - \frac{c(0)}{\Omega} \right] \Phi - \frac{c(0)}{\Omega} [\overline{w} - \frac{c(0)}{2}] - \left[ 1 - \frac{c(0)}{\Omega} \right] \Phi
\]

Thus, the long-run gain is greater than the short-run loss if

\[
-L + \rho G > 0,
\]

where \( \rho \) is the social discount rate across generations.

Inserting (6.6) and (6.7) into (6.8), we get

\[
-\frac{rc(0)}{1 + r} \overline{w} - \frac{(1 - p)(c(p))^2 - (c(0))^2}{2\Omega} - \frac{\Phi}{\Omega} (c(p) - c(0)) + \rho(\beta - 1) \frac{c(0)}{\Omega} \overline{w} > 0,
\]

namely

\[
\beta > \frac{0.5[(1 - p)(c(p))^2 - (c(0))^2] + \Phi(c(p) - c(0))}{\rho c(0) \overline{w}} + \frac{r + \rho + r\rho}{\rho(1 + r)}.
\]

In other words, if the condition (4.2) is satisfied, then (6.8) will be satisfied. \( \square \)
Appendix B: Complementary Simulations

In this appendix we conduct additional simulations aimed at assessing the sensitivity of
the results that were obtained in Section 5. To this end, we fix $p$ at 5% but allow the value
of wage in $F$, $w^f$, to vary. Also, we make a different assumption regarding the evolution
of $\Omega$. The specifications of the remaining parameters are the same as those in Section 5.
Namely,

$$\alpha = 0, \ w^f = 1, \ w^h = 2, \ p = 0.05, \ r = 0.5, \ \Phi = 1.2, \ \Omega = 3 \hspace{1cm} (6.9)$$

$	ilde{w}$ follows a uniform distribution over the domain $[1, 2]$, which implies $\bar{w} = 1.5$.

Varying $w^f$

The logic of the analysis is essentially the same as that in Section 5. The only difference
is that the variables will be a function of $w^f$ instead of $p$. We then derive the following
Table:

<table>
<thead>
<tr>
<th>$w^f$</th>
<th>$l^u$</th>
<th>$F(w^f)$</th>
<th>$u$</th>
<th>$l^v$</th>
<th>$\lambda$</th>
<th>$\eta$</th>
<th>$M$</th>
<th>$d$</th>
</tr>
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<td>2</td>
<td>.83</td>
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<td>.16</td>
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<td>2.79</td>
<td>.29</td>
<td>.55</td>
<td>.05622</td>
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<td>.05693</td>
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<td>.89</td>
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<td>63.61</td>
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<td>.05920</td>
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</tbody>
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Table B1 shows that when the wage gap between the foreign country and the home
country widens, there will be a higher level of educated unemployment and a higher level
of overeducation. For example, when $w^f = 12$ such that the income gap is $\frac{12}{15} = 8$, then the
unemployment rate among the educated is 35% and the proportion of the over-educated
is 63.61%, even though $p = 5\%$. Meanwhile, as the wage gap between the foreign country
and the home country widens, the average cost of educated unemployment, and that of overeducation, will also increase. However, we note that an increase in the wage gap between the foreign country and the home country has little impact on the direct cost of migration or on the number of migrants. This is so because $p$ is at a fixed level no matter what the foreign wage is. Also, as the foreign wage rises, individuals with lower qualifications (for whom the cost of education is higher) will receive education. Consequently, the migrants will increasingly include less qualified individuals. Losing these individuals through migration may not even have negative impacts on the average income in the home country.

The brain gain

Now we rewrite (5.25) as

$$\Omega_t = \Omega_{t-1} - \pi(k_{t-1} - 0.1)$$

where $\pi$ is a positive parameter. In this part, we specify $p_{t-1} = .05$ and $w^f = 5$. Then, from Section 5, we know that

$$k_{t-1} = 0.1529$$

Also, we specify $p_t = .05$. We thus derive the following Table:

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
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</tr>
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<td>21.93</td>
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<td>23.74</td>
</tr>
</tbody>
</table>

Table B2 shows that the results obtained in Table 5 are quite robust.
References


