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Initial Human Capital Distribution and Long Run Income Distribution*

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Abstract

This paper emphasizes the effects of an individual's effort in study and his parental human capital in his human capital formation. It demonstrates the possibility of persistent income inequality even if education is completely free (e.g. completely public). It indicates that the initial parental human capital distribution determines subsequent human capital formation and occupational choice, and hence an economy's long-run income distribution. This study also shows that, as a result of labor market discrimination, minorities are pushed to the two extremes of an economy's income distribution, depending on the level of their initial parental human capital. (JEL: O40, D31)

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1 Introduction

Much recent literature has focused on the interaction between income distribution and economic growth.\(^1\) In particular, a new wave of research has led to an increasing awareness of the non-ergodic property of long-run income distribution. Empirically, this recognition is consistently supported by much evidence.\(^2\) In fact, as summarized by Durlauf (1992), "Granting that the cross-section income distribution exhibits mean reversion, there also exists substantial evidence of persistence in the (lower) tails of the income distribution."

In the existing literature, there are roughly two theoretical approaches to explain this non-ergodic long-run income distribution. One approach is based on the tradition of Becker and Tomes (1979) and Loury (1981), who emphasize that an individual's human capital is determined by his parents' expenditure on his education. For example, Galor and Zeira (1993) show that when credit markets are imperfect and when the expenditure on children's education is indivisible, the economy may be characterized by multiple long-run equilibria and the current distribution of income affects long-run equilibria of individual income.

The other approach, which seems to generate much recent research interest,\(^3\) emphasizes the effects of neighborhood on individuals' human capital formation. Several models of this approach show that communities are formed endogenously and, in equilibrium, low-income people and high-income people will live in different communities. Thus, children in a low income neighborhood will have lower educational attainment, and hence lower earnings because of the lower expenditure on their local school that they attend (e.g. Durlauf 1992); or they will have less motivation to work towards becoming skilled workers because they lack either a social network in the labor market or successful role models in their community (e.g. Benabou 1993).

This paper studies a mechanism that can generate persistent income inequality even


\(^2\)For example, Sawhill 1988; Copper, Durlauf and Johnson (1993); Wilson 1987; and Brittan 1977.

when individuals can get access to the same amount of educational resources\textsuperscript{4} (i.e. when education is completely free) and even when the neighborhood effects are not considered. Then, an extension of the model examines the long-run effects of some social factors, such as labor market discrimination or the lack of social network in the labor markets, on an individual's human capital formation and consequently on income distribution.

In contrast to most previous theoretical studies of intergenerational earnings transmission, this analysis emphasizes the effects of an individual's incentive (or effort) in study and his parental human capital\textsuperscript{5} in his human capital formation. The positive impact of parents' human capital on children's educational attainment is consistently documented by numerous empirical studies.\textsuperscript{6} In fact, several educators (e.g. Sahota (1978: 20), Van der Eyken (1977: 70)) estimate that "about 50 percent of cognitive development of children occurs by the age of three or four." Clearly, as suggested by Osberg (1984), almost all of this early environment is created by one's parents and it is in these early years that a child's basic personality traits such as self-confidence and achievement motivation, which significantly affect his future academic performance, are largely shaped. When a child goes to school, some evidence (e.g. Vernon 1979) shows that the home environment continues to affect his efficiency in learning at school.\textsuperscript{7}

The other factor of human capital formation emphasized in this paper is an individual's incentive (or effort) in study, which has been largely ignored in the economic literature.\textsuperscript{8} The empirical significance of the role of an individual's effort (or incentive) in study in his human capital formation is extensively documented in education and sociology literature.

\textsuperscript{4}We may interpret this assumption as that the state, rather than individual families, is responsible for financing education.

\textsuperscript{5}The externality effect of human capital is highly emphasized in some recent growth literature (e.g. Lucas 1988, Becker et al 1990).

\textsuperscript{6}For example, Coleman et al, 1966; Behrman et al, 1992; Taubman, 1978; and Haveman, 1987 among others.

\textsuperscript{7}It should be noted that the interpretation that parents' human capital affects children's learning efficiency is only to make the model more intuitive. As we will see in the model, the results of the model remain the same as long as parents' human capital affects positively children's human capital formation, no matter how we interpret the exact channel of the influence.

\textsuperscript{8}A notable exception is Glomm and Ravikumar (1992).
This paper develops an overlapping generation model in which individuals live for two periods. In the first period, an individual (a child) is endowed with one unit of time, which he can allocate to consume as leisure or to accumulate human capital.\textsuperscript{10} In this model, since education is a completely free, an individual's only cost of acquiring education is his sacrifice of leisure.

In the second period, an individual (a parent) engages in production. We assume that there are two production technologies\textsuperscript{11} that are both available to all individuals. One is "traditional" (unskilled-intensive) production process, whose input is only physical labor (beyond some "basic" education that every individual may acquire with little cost;\textsuperscript{12} the other is "modern" (skilled-intensive) production process, whose input is efficiency labor (or mental labor, roughly speaking). Every individual is assumed to be endowed with one unit of physical labor in his second period, while the amount of efficiency labor that an individual has is equal to his human capital that he acquires in his first period.\textsuperscript{13} Since an individual can only perform tasks at one of the production technologies at a time, he will choose the occupation (skilled or unskilled) that can yield him higher earnings in the second period, given the amount of human capital he accumulates in the first period.

The basic model of this paper abstracts from the consideration of uncertainty. So, in a

\textsuperscript{9}In fact, this empirical importance is also illustrated by an article on the explanations of the relative economic success of Asian Americans: "...a recent survey of students in San Francisco ... found male Asian-American students spend an average of 11.7 hours a week doing homework, compared with 8.0 hours for whites and 6.3 hours for blacks." (Source: "Why Asians Are Going to the Head of the Class," New York Times (August 3, 1986): EDUC 18-23.)

\textsuperscript{10}An individual's study effort is measured as the sacrifice of his leisure in the model.

\textsuperscript{11}This type of two sector formulation seems to be introduced by Grossman and Helpman (1991).

\textsuperscript{12}For example, a 1990 report by the National Center on Education and the Economy points out that of America's 117m jobs, 40m (34\%) did not require a high-school education (The Economist, August 22, 1992). Since the attendance levels in primary and secondary education are near 100\% in the U.S., these 40m jobs effectively did not require the workers to have any human capital beyond their "basic" education that almost everyone has.

\textsuperscript{13}Our emphasize of formal schooling as the source of individuals' human capital accumulation is consistent with much other existing literature. One may argue that an individual also acquires human capital through learning by doing. However, from the evidence of the age-income profiles of different educational groups (e.g. Willis 1985), it is reasonable to assume that the amount of human capital that an individual acquires at work through either on the job training or learning by doing is an increasing function of his educational attainment. Thus, an individual's earning is ultimately determined by his educational attainment.
world with perfect foresight, an individual’s occupational choice is effectively made in his first period. A rational individual will choose the occupation that can yield him higher lifetime utility. Although education is completely free, due to the differences of parental human capital across families, individuals may differ in their efficiency of learning at school. The more human capital a parent has, the higher his child’s efficiency of learning will be; therefore, the more likely the child will find it optimal to be a skilled worker. This analysis shows that there is a critical level of human capital, such that if an individual’s parental human capital exceeds this level, he will choose to be a skilled worker. Therefore, if a skilled worker’s human capital is above that critical level, his offspring will always choose to be skilled. But if an individual’s parental human capital is below that critical level, he will choose to be unskilled. Also, because human capital (beyond some “basic” education) does not increase the productivity or the wage rate of an unskilled worker, if an individual decides to be unskilled, then he will not forego leisure and spend effort in study.\textsuperscript{14} Thus, his human capital will be even below that of his parents. Consequently, his children will be even lower parental human capital than his and they will therefore also choose to be unskilled. This process will be repeated from one generation to the next, and all of his offspring will choose to be unskilled.\textsuperscript{15} Thus, this paper predicts a very strong correlation of intergenerational occupational choices, which is confirmed by some empirical evidence (e.g. Brittain 1977; Jencks 1979).

Under some reasonable conditions, this analysis shows that individual income will converge to the mean within the group of skilled individuals as well as within the group of unskilled individuals in the long run. Within the higher income group, although an individual with higher parental human capital tends to accumulate more human capital and hence

\textsuperscript{14} For simplicity, this paper assumes away the altruism of parents toward their children. However, as will be discussed in Section 3, this consideration will not change any result of the paper qualitatively.

\textsuperscript{15} If an individual has both a skilled parent and an unskilled parent, his learning efficiency will be a function of some sort of (convex) combination of both of his parents' human capital. So he may or may not choose to be skilled, depending on the levels of his parents' human capital. However, there is much evidence on the positive assortative mating by education (see the survey by Becker (1991)). For example, in the United States, college-educated men are 15 times as likely to marry college-educated women as are men who never completed high school (Becker 1991). Thus, as a first order approximation, we might as well ignore this complication.
to be richer in the short run, his earnings will be closer to the mean than his parents', and therefore the income distribution will exhibit mean reversion within that group. Thus, the economy is characterized by multiple steady state equilibria. In the long run, individuals will be segmented into two types of dynasties. Each individual in rich dynasties acquires a high level of education and becomes a skilled worker, while each individual in poor dynasties will work as an unskilled worker. The economy's steady state equilibrium is path dependent. It is determined by the initial distribution of human capital of this economy. Thus, the model predicts that income may not converge globally to the mean even if education is completely free and even if neighborhood effects are not considered. It complements other models to show that income distribution may be non-ergodic in the long-run.

In this model, since education is completely free, parents' earnings per se do not affect their children's earnings. However, because parents' human capital affects children's learning efficiency at school, which in turn affects children's human capital formation, and because an individual's earnings are associated with his human capital, intergenerational earnings are correlated. Therefore, when education is completely public (i.e. free), this analysis shows that it is an economy's initial (parental) human capital distribution, more than its initial income distribution, that determines the human capital distribution and hence the income distribution of the next generation. The initial human capital distribution of an economy may be unequal across families due to various historical factors. For example, individuals might migrate (voluntarily or involuntarily) from societies with different levels of human capital; or individuals may have different human capital because education was private and income distribution was unequal (e.g. Loury 1981); or individuals might not be able to go to school because they belong to the oppressed races and racial discrimination was severe. Therefore, the model provides an explanation for the observation that the poverty of various immigrant groups arriving in the U.S. in the early 20th century is relatively transitory while

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16 This result seems to be related to Lundberg and Startz (1993), who apply the analogy of "two nations" to describe race relations in the U.S.

17 Clearly, the result of the model will be reinforced if we consider that higher human capital individuals usually live in better neighborhoods and spend more on their children's education (since they have higher income).
inner-city poverty in the U.S. is persistent.\textsuperscript{18}

This model has several interesting policy implications. First, economists as well as policy-makers have long been puzzled by the fact that many anti-poverty programs that increase expenditures on poor children's education have been far less effective than had been expected (see the survey by Sawhill (1988)).\textsuperscript{19} Emphasizing the roles of an individual's incentive in study and his parental human capital in his human capital formation, this model sheds light on the puzzle by showing that income inequality may persist even if every individual has access to the same amount of educational resources. Second, since the model implies that the increase of an individual's learning efficiency makes him more likely to spend enough effort in study to be skilled, it also helps explain the relative success (e.g. Halpern 1982; Currie and Thomas 1993) of some types of anti-poverty programs that intended to increase poor children's learning efficiency through early childhood day-care and special compensatory preschool education program, such as Head Start.\textsuperscript{20} Finally, this study indicates that the timing of educational subsidy matters. When an economy's human capital distribution becomes sufficiently unequal (possibly resulting from very unequal initial income distribution), in contrast to most previous studies, this model shows that the persistence of income inequality will be robust to temporary wealth transfers across different families by government policy intervention (e.g. tax or subsidy).

The second part of this paper extends the model to the situation in which an individual's

\textsuperscript{18}It can be explained that new immigrants came to the U.S. with (significantly) higher level of educational attainment than the minority individuals living in the American inner cities, who had been deprived of their educational opportunities by many generations' severe discrimination. Although the new immigrants were poor because of the language barrier or because their human capital might be too specific to their home countries to be applied in the U.S., their children might have high efficiency in learning at school (because of their high parental human capital) and might become economically successful through acquiring good education.

\textsuperscript{19}In fact, Sawhill (1988, p 1085) argues: "...Although economists are often criticized for devoting too much time to theory at the expense of empirical information, I believe that where the distribution of income and poverty are concerned just the opposite is the case. We are swamped with facts about people's income and about the number and composition of people who inhabit the lower tail, but we don't know very much about the process that generates these results."

\textsuperscript{20}However, it should be noted that the effect of these programs such as Head Start may be limited because of the limited fund of the government, and because these programs are not perfect substitute for the home environment, and because there may exist significant moral hazard problem since the teachers of these programs do not teach their own children.
income is determined by some social factors as well as his human capital. In particular, it analyzes the long-run effects of labor market discrimination.

Suppose that due to labor market discrimination or the lack of social network in the labor markets, an individual who would be a skilled worker may not be able to get a skilled job and may have to work as an unskilled worker. This analysis shows that this adverse effect will make more individuals of a minority group choose to be unskilled. Meanwhile, even if he *tries* to be a skilled worker, an individual with the expectation that he will have less chance of getting a skilled job will have less incentive to study. Consequently, skilled workers from minority groups will accumulate less human capital and earn at a lower wage rate in the long run even if they are paid according to their productivity when they are hired as skilled workers. Finally, the model implies that even transitory labor market discrimination may have permanent effects on minority individuals’ occupational choices. Specifically, a minority individual may choose to be unskilled only because of the anticipated discrimination in the future skilled labor markets. Thus, he will spend little effort on school work. Consequently, his children will have low parental human capital and hence low learning efficiency at school. Therefore, they may continue to choose to be unskilled even if labor market discrimination no longer exists. Therefore, we provide a model that is consistent with Wilson’s (1987) insight that “historic discrimination is more important than contemporary discrimination in understanding the plight of the black underclass.”

However, suppose a minority individual can escape discrimination by accumulating human capital up to a certain high level and by becoming “highly” skilled. This possibility exists since superior human capital, such as outstanding academic performance, can distinguish such an individual from other minority group members, thus avoiding “statistical discrimination” (Arrow 1973), or his superior human capital can situate him in a position (or profession) not easily substituted by others. Under such an additional consideration, this study shows that individuals whose parental human capital exceeds another certain threshold level will find it optimal to spend enough effort in study to be “highly” skilled. If

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21This point is also made in Lundberg and Startz (1993) from a different perspective.
the minimal level of human capital to be "highly" skilled exceeds this threshold level, then a "highly" skilled worker's child will always have the parental human capital that is above the threshold level. Therefore, all of his offspring will choose to be highly skilled.

Thus, this analysis shows that the long run effect of labor market discrimination is that minorities are pushed to the two extremes of an economy's income distribution, depending on their parents' educational level. This argument is illustrated by the relative economic success of many Asian Americans versus the persistent poverty of many African Americans. Consequentially, an interesting testable prediction of this model is that rich minority individuals' income will decrease and poor minority individuals' income will increase when discrimination against minorities is reduced.

In what follows, Section 2 establishes the basic analytic framework; Section 3 investigates the relationship between an individual's parental human capital and his human capital formation such that his occupational choice; Section 4 analyzes the multiple long-run equilibria of the dynamics of individuals' human capital accumulation of an economy; Section 5 extends this model to explore the influence of some social factors (e.g. labor market discrimination) on an individual's human capital formation and his occupational choice; Section 6 offers the conclusion; the mathematical proofs of this model are mostly provided in the appendix.

2 The Basic Analytic Framework

Consider individuals who operate in a two period overlapping generations world. Every individual belongs to a "family," where he is a child in his first period and becomes a parent in his second period. Each individual has one parent and one child, so there is no population growth in this economy.

In his first period, an individual is endowed with one unit of divisible time, which he can

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22It should be noted that many African Americans may have low parental human capital only because their predecessors faced very severe discrimination and could not go to school, consequently their parental human capital decreased over time. But when the massive Asian immigrants came to the U.S. after the second world war (particularly after the civil rights movements), they faced much less discriminations.
either consume as leisure or use to accumulate human capital (i.e. spend effort in study). We assume that an individual obtains utility from leisure, or equivalently, gets disutility from spending effort in study. Let \( e \) denote the amount of effort (time) that an individual spends on his study, \(-\nu()\) the disutility function from spending effort in study. We assume that \( \nu() \) satisfies the neoclassical property, which is formally captured by the following assumption.

**Assumption 1**

\[
\nu'(e) > 0, \nu''(e) > 0, \forall e > 0; \nu(0) = \nu'(0) = 0, \lim_{e \to 1} \nu(e) = \lim_{e \to 1} \nu'(e) = \infty
\]

**Remark:** It is easy to verify that the two inequalities of the assumption is equivalent to assume that the utility function of leisure is strictly increasing and strictly concave, while the assumption of the first derivative of \( \nu() \) at \( e = 0 \) and \( e = 1 \) is equivalent to assume that the utility function of leisure satisfy the Inada condition. Also, we normalize an individual's minimal amount of effort in study and the corresponding disutility to zero. But we may interpret that an individual spends zero effort in study as that he receives (with little cost) some "basic" education that everyone in the society has.\(^{23}\) Finally, we assume the disutility that an individual would get if he does not take any leisure and spends all of his time on study to be very large, and formally, to be negative infinity.

As we will see, an individual's effort in study plays a very important role in his human capital formation in the model. Meanwhile, it also affects the individual's utility (or disutility). In fact, This emphasis that preferences can play a role in human capital acquisition is also stressed by a recent contribution by Pollak (1994). Also, it might be more realistic to assume that an individual's disutility (or cost) of study decreases if he has a better home environment (i.e. higher parental human capital). However, as will be apparent, this consideration will only reinforce our results. So, for simplicity, we might as well ignore this complication.

Individuals are assumed to have access to the same amount of educational resources, namely, education is completely free. So an individual's only cost of acquiring human capital

\(^{23}\)It should be noted that it is possible that an individual stays at school but spends no effort in study. There is much anecdotal evidence that many high school graduates in New York city are illiterate although they have stayed in school for 12 years, which indicates that they spent little effort in study at school.
is his sacrifice of leisure. In this paper, we emphasize that an individual’s effort in study and his parental human capital are the only factors of an individual’s human capital formation function. Let \( e_t, H_t \) denote the amount of effort in study, and the amount of human capital of an individual of generation \( t \) respectively, then the human capital production function is defined as,

\[
H_{t+1} = h(H_t, e_{t+1})
\]  

(1)

We assume that the two inputs of the production function are complementary, and the production function is strictly increasing and strictly concave with respect to its variables, and satisfies the Inada condition when either of its input approaches its limit. Then, formally, we have,

**Assumption 2**

\[ h_1(H, e) > 0, h_2(H, e) > 0, h_{12}(H, e) > 0, h_{11}(H, e) < 0, h_{22}(H, e) < 0, \forall \alpha, \epsilon \]

\[
\lim_{H \to \infty} h_1(H, e) = \lim_{\epsilon \to 1} h_2(H, e) = 0
\]

Also, we assume that an individual does not consume any material good in his first period. This can be explained that when an individual (a child) is a student at school, his consumption is provided by his parents. When we assume that he cannot borrow against his future income, which is typically the case, his consumption can be regarded as exogenously fixed.

In his second period, an individual earns income from engaging in the production of material goods. In this period, we assume that an individual obtains utility only from the consumption of material goods. The utility function, which is denoted by \( u() \), is assumed to be strictly increasing and strictly concave and twice differentiable. For simplicity, we assume that there is no bequest in this economy.\(^{24}\) Thus, an individual will consume all of his income in his second period.

\(^{24}\)This assumption was also made in Becker and Tomes (1979), Loury (1981) and many other studies on intergenerational earnings mobility.
We assume individuals operate in a small open economy\(^{23}\) in a one-good world. The good can be produced by two constant return to scale technologies. One is "modern" production technology, which is human capital (or skilled labor) intensive; the other is "traditional" production technology, which is physical (or unskilled) labor intensive. To capture this property formally, the simplest way is to assume that the only input in the modern (skilled) sector is efficiency labor (or mental labor, simply speaking), while the only input in the traditional (unskilled) sector is physical labor.

Specifically, the production function of the modern (skilled) technology is described by:

\[ Y_t^s = w_s L_t^s \]  

(2)

where \( Y_t^s \) and \( L_t^s \) are the output in this sector and the efficiency labor input of the whole economy at time \( t \) respectively, \( w_s \) is the marginal productivity of efficiency labor.

The production function of the traditional (unskilled) technology is described by:

\[ Y_t^n = w_n L_t^n \]  

(3)

where \( Y_t^n \) and \( L_t^n \) are the output in this sector and the unskilled labor force of the whole economy at time \( t \) respectively, \( w_n \) is the marginal productivity of physical labor.\(^{26}\)

The markets for both efficiency labor and physical labor are perfectly competitive. Thus, the wage rate per unit of efficiency labor is \( w_s \), while the wage rate per unit of physical labor is \( w_n \).

These two production technologies are available to every individual. However, since an individual can only perform tasks at one of the two sectors at a time, he may choose to work either as a skilled or an unskilled worker, but not both. Every individual is assumed to be endowed with one unit physical labor in his second period, while the amount of efficiency

\(^{23}\)This small open economy may refer to a small country of the world economy, or a city of a large country, etc.

\(^{26}\)This simple formulation seems to be introduced by Grossman and Helpman (1991) and is also used in some other recent growth literature. However, it is easy to verify that we can obtain qualitatively the same results as long as we assume that human capital is used less intensively in the traditional sector. It is only for the maximal technical simplicity that the model, as in Grossman and Helpman (1991), assumes that physical labor is the only factor of production in the traditional sector.
labor that an individual has is equal to his human capital he acquires in his first period. Thus, for an individual with $H$ amount of human capital, if he takes a skilled job, his income will be $w_sH$; if he takes an unskilled job, his income will be $w_n$. Thus, given the amount of human capital he has, $H$, an individual will choose to be skilled in his second period if and only if,

$$w_sH \geq w_n$$

(4)

From (4), we then have the following lemma.

**Lemma 1** Given the amount of human capital he has, $H$, an individual will choose to work in a skilled job in his second period if and only if

$$H \geq w_n/w_s$$

**Remark** This lemma is illustrated in Figure 1,

Figure 1 is about here

where we can see that the combination of the two production technologies results in some locally increasing return to skill in an individual's earnings function. This theoretical result of the existence of locally increasing return to skill is confirmed by some recent empirical evidence (see the survey by Levy and Murnane, 1992).

### 3 Parental Human Capital and Occupational Choice

In this section, we will analyze an individual's decision on being skilled or being unskilled. The basic model of this paper abstracts from the consideration of uncertainty. So, in a world of perfect foresight, an individual's occupational choice is effectively made in his first period. A rational individual will choose the occupation that can yield him higher lifetime utility. So, firstly, we will investigate an individual's optimal choice of the amount of effort in study and hence his maximal utility in either being skilled or being unskilled.
When an individual (of generation $t+1$) chooses to be unskilled, his income is certain at $w_n$, and his intertemporal utility function is,

$$u(w_n) - v(e_{t+1})$$  \hspace{1cm} (5)

The constraint is,

$$0 \leq e_{t+1} \leq 1$$ \hspace{1cm} (6)

Since an individual gets disutility from spending effort in study, obviously, the optimal solution to (5) is to set $e_{t+1} = 0$. Thus, we have the following lemma,

**Lemma 2** When an individual chooses to be an unskilled worker in his second period, he will spend zero amount of effort in study in his first period.

When an individual chooses to be a skilled worker, his intertemporal utility function is,

$$U \equiv u(w_sH_{t+1}) - v(e_{t+1})$$

$$= u[w_s h(H_t, e_{t+1})] - v(e_{t+1})$$ \hspace{1cm} (7)

The constraint is also (6). So the first order condition is,

$$w_s u'[w_s h(H_t, e_{t+1})] h_2(H_t, e_{t+1}) - v'(e_{t+1}) = 0$$ \hspace{1cm} (8)

It should be noted that the corner solutions $e_{t+1} = 0$ and $e_{t+1} = 1$ are excluded by $v'(0) = 0$ and $\lim_{e \to 1} v'(e) = \infty$ (Assumption 1) respectively.

Now, we have the following lemma (The proofs of lemmas, theorems, and corollaries are all provided in the appendix if not provided in the text.)

**Lemma 3** If an individual chooses to be a skilled worker, given the amount of his parental human capital, his optimal choice of the amount of effort in study exists and is unique.

Let the optimal solution be $e_{t+1}^*$. By Lemma 3, it is a function of $H_t$. So we can define it as

$$e_{t+1}^* = e(H_t)$$
1. Totally differentiating (8) with respect to $e_{t+1}$ and $H_t$, we get,

$$
\frac{de_{t+1}}{dH_t} = \frac{u_s^2 u'' h_1 h_2 + w_s u'h_{12}}{u'' - u_s^2 u'' h_2^2 - w_s u'h_{22}}
$$

(9)

The denominator of the right hand side of (9) is obviously positive. The first item of the numerator is negative, which may be regarded as the “income effect” when divided by the denominator; the second item of the numerator is positive, which may be regarded as the “substitution effect” when divided by the denominator. However, no matter what the net effect is, we have the following proposition.

**Theorem 1** If an individual chooses to be a skilled worker, then

$$
\frac{dH_{t+1}}{dH_t} > 0
$$

**Remark:** This theorem indicates that if he chooses to be skilled, an individual with higher parental human capital will accumulate more human capital and hence get higher income even if his higher parental human capital might make him study less hard (i.e. the “income effect” is greater than the “substitution effect”). Thus, this theorem complements the previous studies (e.g. Becker and Tomes 1979, Loury 1981), by showing that intergenerational earnings are positively correlated even if education is completely free.

Now we define,

$$
G \equiv u(w_s h(H_t, e_{t+1}^*)) - v(e_{t+1}^*) - u(w_n)
$$

$$
= u[w_s h(H_t, e(H_t))] - v(e(H_t)) - u(w_n)
$$

Clearly, $G$ measures the difference of an individual’s intertemporal utility of being skilled and being unskilled. So an individual will choose to be skilled if and only if $G \geq 0$.28

The following lemma discusses the relationship between $G$ and its variables.

27The “income effect” and “substitution effect” in this context are interpreted that as an individual’s parental human capital increases, his efficiency of study will increase. Consequently, his income will increase (holding other variables constant) and his opportunity cost of taking leisure will also increase. An individual will consume more leisure (study less hard) as his income increases (“income effect”), while he will consume less leisure (study harder) as the (opportunity) cost of taking leisure increases (“substitution effect”).

28For simplicity of exposition, we might as well assume that an individual will choose to be skilled as long as he can be weakly better off by being skilled than being unskilled.
Lemma 4 \( G \) is a strictly increasing function of \( H_t \) and \( w_s \), and is a strictly decreasing function of \( w_n \).

Remark: This lemma indicates that (1) An individual is more likely to choose to be skilled as the wage rate for skilled workers rises or his parental human capital increases. (2) An individual is more likely to choose to be unskilled as the wage rate for unskilled workers rises.

Now we add an assumption that serves as a sufficient condition of the next theorem.

Assumption 3 (1) \( h(0,1) < w_n/w_s \); (2) There exists a large number \( N \), such that \( h(N,0) > w_n/w_s \).

Remark: This assumption means that if an individual's parental human capital is zero (namely, at the normalized minimal level), his learning efficiency at school will be so low that he would not be able to accumulate enough human capital to earn more than an unskilled worker's wage in his second period even if he does not consume any leisure and only study in his first period. On the other hand, if an individual's parental human capital is sufficiently high, he would be able to accumulate enough human capital to earn more than an unskilled worker's wage even if he just spent the normalized minimal amount of effort in study.

Based on the above description, now we have the following theorem.

Theorem 2 Under the above stated assumptions, there exists a critical value \( H^c \), such that an individual will choose to be skilled if and only if his parental human capital is greater or equal to \( H^c \). Besides, \( H^c \) is a strictly increasing function of \( w_n \), and a strictly decreasing function of \( w_s \).

Remark: This theorem establishes the relationship between an individual's parental human capital and his occupational choice. The higher his parental human capital is, the more likely an individual will choose to be skilled. Meanwhile, it also shows that an individual will be less motivated to be skilled if the wage rate for unskilled workers increases or the one
for skilled workers decreases. Thus, this theorem, together with Lemma 4, indicates that
governmental transfers or subsidy to the wages of unskilled workers has long-run disincentive
effect by discouraging some individuals with low parental human capital to accumulate
enough human capital to become skilled workers. This implication is consistent with some
recent empirical evidence (see the Survey by Moffitt (1992)).

Now we will show that under two more assumptions below, individuals will be segmented
into two groups of dynasties in the long run.

Assumption 4

\[ h(H_t, 0) < H_t, \forall H_t (H_t \neq 0); h(0, 0) = 0 \]

Remark: In this model, we normalize an individual’s minimal level of human capital to zero.
The above assumption means that no matter what kind of family background an individual
has, if he does not study at all, his human capital will be less than his parents’ (e.g. because
of the deterioration of human capital). If an individual’s parental human capital is zero,
and he does not study at all, his human capital will also be at the minimal level.

Assumption 5

\[ h(H^c, e(H^c)) > H^c \]

Remark: This assumption means that when an individual’s parental human capital is just
at the threshold level that make him choose to be skilled, he will accumulate more human
capital than that threshold level. It should be noted that because \( h(H^c, e(H^c)) \geq w_n / w_s \)
(by Lemma 1), the above assumption is weaker than the assumption \( w_n / w_s > H^c \), which
means that a skilled individual’s human capital is always above the critical value of human
capital \( H^c \). Also, Corollary 2 in next section will show that Assumption 5 is necessary to
rule out the possibility of the unrealistic implication of the model that all individuals will
be unskilled in the long run.

Now, we have the following corollary.
Corollary 1—Under the above stated assumptions, there exist two groups of dynasties in the economy, where the members of one group will always be skilled, while the members of the other group will always be unskilled.

The proof of this corollary has some interesting economic intuition. So it is provided in the text.

Proof of Corollary 1: If an individual’s parental human capital $H_t$ satisfies $H_t \geq H^c$, by Theorem 2, this individual will choose to be a skilled worker. By Theorem 1 and Assumption 5, we have

$$H_{t+1} = h(H_t, e(H_t)) \geq h(H^c, e(H^c)) > H^c$$

So, again by Theorem 1, this individual’s child will also choose to be a skilled worker. This process will be repeated from one generation to the next, and all of this individual’s offspring will choose to be skilled.

On the other hand, if an individual’s parental human capital $H_t$ is less than $H^c$, namely, $H_t < H^c$, by Theorem 1, this individual will choose to be an unskilled worker. In this case, by Lemma 2, he will spend zero (i.e. minimal) amount of effort in study. Then, by Assumption 4, his human capital will be

$$H_{t+1} = h(H_t, 0) < H_t < H^c$$

Thus, his child will also choose to be an unskilled worker and will spend zero effort in study. This process will be repeated from one generation to the next, and all of this individual’s offspring will choose to be unskilled.

Q.E.D.

Remark of Corollary 1: There are three comments on this corollary. First, the essence of this corollary is that it predicts a very strong correlation of intergenerational occupational choices. However, it should be noted that the model implies there is perfect intergenerational transmission of occupational choices only because it abstracts from the consideration of uncertainty. If the model is extended to take into account some random factors such as individuals’ innate ability or market luck, it would predict that children whose parents are
skilled are (much) more likely to be skilled than those whose parents are unskilled. This prediction, however, is exactly the essence of the corollary.

This prediction is confirmed by some empirical evidence (e.g., Jencks 1979, Brittain 1977). In particular, based on his eleven panel studies on American men, Jencks (1979, p214) argues that "All aspects of family background explained about 48% of the variance in mature men's occupational statuses (and) the most important single measured background characteristic affecting a son's occupational status is his father's occupational status."

The second comment on the corollary is that although parental human capital plays a critical role in an individual's human capital formation and consequently his occupational choice, explicit altruism is assumed away from the model. However, because the co-existence of modern and traditional production technologies yields some local non-convexity of an individual's earning function of human capital, it is not difficult to verify that we can obtain the same qualitative results of the model when altruism is considered. To see this, suppose we extend the model to assume that parents obtain utility from their human capital itself, considering its externality effect on their children human capital formation. If we assume this new utility function is concave, from (7), the combination of \( u(w_t H_{t+1}) \) and the new utility function will have the same qualitative property as \( u() \) with respect to its variable \( H_{t+1} \). So, from the proofs (most of which are provided in the appendix), it is clear that this consideration will not change any of our results qualitatively.

Finally, if an individual has both a skilled parent and an unskilled parent, his learning efficiency will be a function of some sort of (convex) combination of both of his parents' human capital. So he may or may not choose to be skilled, depending on the levels of his parents' human capital. However, there is much evidence on the positive assortative mating by education (see the survey by Becker (1991)). For example, in the United States, college-educated men are 15 times as likely to marry college-educated women as are men who never completed high school (Becker 1991). Thus, as a first order approximation, we might as well ignore this complication that an individual may have both a skilled parent and an unskilled parent.
In this model, we see that an individual’s parental human capital fully determines his occupational choice. Now, let $D_0$ be the distributional function of the initial (parental) human capital (at time 0) in this economy. Since each individual has one child and one parent, the total number of the workforce in this economy is constant over time, and we denote it by $L$. Then, by Corollary 1 and its proof, the number of unskilled labor in this economy at any time $t$ is,

$$L^n = \int_0^{H^c} dD_0(H_t)L = D_0(H^c)L$$  \hspace{1cm} (10)

and the number of skilled labor is,

$$L^s = \int_{H^c}^\infty dD_0(H_t)L = (1 - D_0(H^c))L$$ \hspace{1cm} (11)

Thus, when the initial distribution of parental human capital neither below $H^c$ nor above $H^c$ is trivial, individuals will be segmented into two groups of dynasties in the long run.

4 Long Run Equilibria

In this section, we will analyze the long run equilibria of the evolution of the human capital distribution and hence the income distribution of the economy. Firstly, the steady state of the dynamics of individuals’ human capital of a dynasty is defined as follows.

**Definition 1** The dynamics of individuals’ human capital of a dynasty is in steady state (at time $t$) if

$$H_t = H_{t+1} = H_{t+2} = .....$$

From (1), it is easy to see that this definition implies that the dynamics of individuals' effort in study of a dynasty is also in steady state, namely,

$$e_t = e_{t+1} = e_{t+2} = .....$$

Now, the following two theorems characterize the properties of the steady states of individuals’ human capital of both groups of dynasties that we described above.
Theorem 3 Under the above stated assumptions, the unique and stable steady state of individuals' human capital of the group of the dynasties who choose to be unskilled is zero.

Remark: The intuition behind this theorem is that because an individual's human capital does not increase his productivity and hence his income when he chooses to be unskilled, individuals of a dynasty who chooses to be unskilled will not choose to spend effort in study and invest in human capital. So individuals' human capital of the dynasty will "depreciate" over time to the minimal level. This theorem indicates that individuals whose parents are unskilled may not choose to be skilled even if the skilled-intensive sector experiences growth (i.e. \( w_s \) rises), since the parental human capital of unskilled dynasties becomes increasingly smaller.\(^{29}\)

Theorem 4 Under the stated assumptions, for the group of dynasties who choose to be skilled, (a) there always exists at least one locally stable steady state of individuals' human capital; (b) there exists a unique and globally stable steady state of individuals' human capital if the following inequality is satisfied,

\[
(1 - h_1(H_t, e_{t+1}))h_{22}(H_t, e_{t+1}) + h_2(H_t, e_{t+1})h_{12}(H_t, e_{t+1}) \leq 0, \forall e_{t+1}, H_t (H_t \geq H^c) \tag{12}
\]

(c) if inequality (12) is satisfied,

\[
\frac{dH_{t+1}}{dH_t} < 1
\]

Remark: (1) When \( h_1, h_2 \) and \( h_{12} \) are small enough such that (12) can be satisfied, individuals' human capital and hence income will converge within the group of the dynasties who choose to be skilled. In the short run, although the child from richer family will still be richer (by Theorem 1), his human capital (hence earnings) will be closer to the mean than his parents', and therefore income will converge across generations within the skilled dynasties.\(^{30}\) (2) If (12) is not satisfied, there may be multiple equilibria of individuals' human capital within the skilled dynasties, which implies income inequality may persist even

\(^{29}\)However, it should be noted that individuals' earnings of an unskilled dynasty are stationary at the level of unskilled labor income \( w_u \) over time despite the diminishing human capital of the individuals of the dynasty.

\(^{30}\)This result is consistent with many previous studies of intergenerational earnings mobility. But different from many of these studies, the income convergence in this model is "conditional".
within the skilled dynasties. These two cases are illustrated in Figure 2a and Figure 2b respectively.

Figure 2a and Figure 2b are about here

Thus, in either case, we have provided a mechanism showing that when the initial human capital distribution is sufficiently unequal, income distribution will not converge to the mean globally even if education is completely free.

However, since most empirical studies seem to indicate that income distribution exhibits mean reversion when the bottom of an economy's income distribution is excluded, we will focus on the situation where the steady state of individuals' human capital of the group of skilled dynasties is unique. Let \( H^* \) denote the level of an individual's human capital (of skilled dynasties) in equilibrium, then a skilled individual's income will converge to \( w_s H^* \) in the long run. Thus, the above two theorems indicate that the long run wage inequality between these two group of dynasties is

\[
 w_s H^* - w_n
\]

Therefore, we have shown that when the initial human capital distribution is sufficiently unequal, income disparity of an economy may persist even if every individual in the economy can get access to the same amount of educational resources (i.e. education is completely free). Therefore, the model complements the previous studies (e.g. Galor and Zeira 1993, Benabou 1993, and Durlauf 1992) in a different angle to predict that the dynamic process of the income distribution of an economy may be nonergodic.

This model has several interesting policy implications. First, economists as well as policy-makers have long been puzzled by the fact that many anti-poverty programs that increase the expenditures on the public schools that poor children attended have been far less effective than had been expected (see the survey by Sawhill (1988)). Emphasizing the roles of an individual's incentive in study and his parental human capital in his human capital formation, this model sheds light on this puzzle by showing that income inequality
may persist even if education is completely free, namely, every individual has access to the same amount of educational resources.31

Second, since the model implies that the increase of an individual’s learning efficiency makes him more likely to spend enough effort in study to be skilled, it also helps explain the relative success (e.g. Currie and Thomas 1993; Halpern 1982) of some other types of anti-poverty programs that intended to increase poor children’s learning efficiency through early childhood day-care and special compensatory preschool education program, such as Head Start. However, it should be noted that the effect of these programs such as Head Start may be limited because these programs may not be the perfect substitute of the home environment in determining an individual’s learning efficiency at school, and because there may exist significant moral hazard problem in these programs since the teachers of these programs do not teach their own children, etc. Thus, these programs may only be able to partially compensate poor children’s low parental human capital to increase their learning efficiency. When an individual’s parental human capital is sufficiently low, the increase of their learning efficiency may not be large enough to change their choice to be unskilled. Consequently, they will spend little effort in study and will have low educational attainment. In fact, this theoretical prediction is supported by a recent empirical contribution by Currie and Thomas (1993), who find that positive and persistent effects of participation in Head Start on the test scores of white and Hispanic children, but no effects on the test scores or school attainment of African American children. As suggested by Currie and Thomas (1993), this result can be explained that African American children may have the most disadvantaged family background.32

Finally, this study indicates that the timing of educational subsidy matters. When an economy’s human capital distribution becomes unequal (possibly resulting from unequal

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31In fact, part of the basic idea of the model is also suggested by Sawhill (1988, p 1097): “...another possibility (why poverty in the U.S. is persistent) is that a great deal of human capital investment goes on within the family and that institutions such as schools have little hope of offsetting the powerful influence of home environment....” Thus, we provide a model that is consistent with Sawhill’s suggestion.

32For example, as suggested by Currie and Thomas, African-American mothers are more likely to single mothers. So they have to spend more time to work and less time educating their children. Also, African Americans are likely to have the lowest human capital because the past severe discrimination in school and in skilled labor markets has made them have little incentive to accumulate human capital for generations.
initial income distribution), in contrast to most previous studies, this model shows that the persistence of income inequality will be robust to temporary wealth transfers across different families by government policy intervention (e.g. tax or subsidy).

Now, the following corollary (of Theorem 4) shows that Assumption 5 is necessary to guarantee the existence of multiple long-run equilibria. If it is violated, individuals in this economy might all be unskilled in the long run.

**Corollary 2** If \( h(H^e, e(H^e)) < H^e \) and if (12) is satisfied, the only steady state of individuals' human capital in this economy is that \( H = 0 \).

**Remark:** Because it is unrealistic that no individual will invest human capital in the long run in an economy where education is completely public (i.e. free), this corollary provides another justification for Assumption 5.

Finally, from (10) and (11), we can get the long-run per capita GNP as,

\[
D_0(H^e)w_n + (1 - D_0(H^e))w_sH^e
\]

Therefore, the initial distribution of human capital determines the long-run well-being of the individuals in this economy. An economy with more equal distribution of initial human capital will tend to have higher average living standard in the steady state.

5 Long-Run Effects of Labor Market Discrimination

In this section, we will extend the basic model to the situation where an individual's income may not be completely determined by his human capital. The purpose of this extension is to examine the long run effects of some social factors, such as labor market discrimination or the lack of social network in the labor markets, on individuals' human capital formation and consequently the long run income distribution of an economy.

Suppose that due to labor market discrimination or the lack of social network in the labor markets, an individual who would be a skilled worker may not be able to get a skilled job and may instead have to work as an unskilled worker. We assume that this is the only
discrimination in this economy, \(^{33}\) and there is no discrimination in the market for unskilled labor. \(^{34}\)

Suppose the probability that a minority individual can be employed as a skilled worker is \(p\) \((0 \leq p \leq 1)\). Then, \(p\) characterizes the degree of discrimination in this economy. The less \(p\) is, the more discrimination there is in the labor markets.

In this case, if an individual (of generation \(t + 1\)) from a minority group chooses to be unskilled in his second period, his intertemporal utility function is the same as we described earlier, and his optimal choice in his first period is still not to invest in any human capital. If he chooses to try to be skilled, however, his intertemporal utility function becomes,

\[
V \equiv pu(w_sH_{t+1}) + (1 - p)u(w_n) - v(e_{t+1})
= pu[w_s h(H_t, e_{t+1})] + (1 - p)u(w_n) - v(e_{t+1})
\] (14)

Then the first order condition is,

\[
p w_s u'[w_s h(H_t, e_{t+1})] h_2(H_t, e_{t+1}) - v'(e_{t+1}) = 0
\] (15)

Similar to Lemma 3, we can show that an individual’s optimal choice of the amount of effort in study exists and is unique if he chooses to be skilled. We denote it by \(e^p_{t+1}\). Since it is a function of \(H_t\), we define it as,

\[e^p_{t+1} = e^p(H_t)\]

Also, similar to Theorem 2, we can show that there exists a critical value \(H^{cc}\), such that a minority individual will choose to be skilled if and only if the level of his parental human capital is greater or equal to \(H^{cc}\). Moreover, we have the following theorem.

**Theorem 5** \(H^{cc}\) is a strictly decreasing function of \(p\). In particular, when \(p < 1\),

\[H^{cc} > H^c\]

\(^{33}\)In fact, at least in most developed countries, explicit discrimination by paying minority workers less wage rate than other workers doing the same job is prohibited by law. Besides, a worker would have less incentive to work if he thinks that his employer does not pay him the “fair” wage rate (Akerlof and Yellen, 1990). Thus, most discrimination in labor markets takes the form that we assumed. In fact, this assumption is justified by a recent analysis of Coate and Loury (1993) when discrimination exists in labor markets.

\(^{34}\)Otherwise, there would be involuntary unemployment in this economy. Thus, this assumption maintains that labor markets clear in a competitive economy.
Remark: This theorem implies that an individual facing uncertainty of being able to be employed as a skilled worker (due to labor market discrimination or the lack of social network in the labor markets) is less likely to choose to spend enough effort in study and try to be skilled. The more discrimination there is in the labor market (e.g. the less $p$ is), the less incentive a minority individual will have to accumulate enough human capital to try to be skilled. Meanwhile, similar to Theorem 3, we can show that the unique and stable steady state of individuals’ human capital of the group of the unskilled dynasties is zero. Thus, this result and Theorem 5 indicate that even transitory labor market discrimination may have long run effects on minority individuals’ human capital formation and occupational choices. Specifically, a minority individual may choose to be unskilled only because of the anticipated discrimination in the future skilled labor market. Thus, he will spend little effort on school work. Consequently, his children will have low parental human capital and hence low learning efficiency at school. Therefore, they may continue to choose to be unskilled even if labor market discrimination no longer exists. Therefore, the model is consistent with Wilson’s (1987) insight that “historic discrimination is more important than contemporary discrimination in understanding the plight of the black underclass.”

Besides, even if a minority individual tries to be skilled, as shown in the following theorem, he will have less incentive to accumulate human capital.

Theorem 6 Given his parental human capital $H_t$, a minority individual’s incentive in study (and hence his human capital) is a strictly increasing function of $p$. In particular, when $p < 1$,

$$e^p(H_t) < e(H_t)$$

Remark: Labor market discrimination may discourage a minority individual to accumulate human capital even if he tries to be skilled. And the more discrimination there is (e.g. the less $p$ is), the less incentive a minority individual will have to accumulate human capital.

Now, similar to Corollary 1, Theorem 4, we can show that (1) If $h(H^c, e^p(H^c)) > H^c$, then there exist two groups of dynasties (among the minority groups) in this economy,
where one group will always choose to try to be skilled, while the other group will always
be unskilled. (2) The steady state of individuals' human capital of the group of the skilled
dynasties will be unique and stable when (12) is satisfied. Now, suppose (12) is satisfied.
let $H^d$ be this unique steady state. Then we have following theorem.

**Theorem 7** If (12) is satisfied, then $H^d$ is a strictly increasing function of $p$. In particular,
when $p < 1$,

$$H^d < H^*$$

**Remark:** This theorem indicates that because of the disincentive effect of labor market dis-
iscrimination on minority individuals' human capital formation, skilled workers from minority
groups will earn at a lower wage in the long run even if they are paid for their productivity
when they are employed as skilled workers. Meanwhile, the more discrimination there is
(e.g. the less $p$ is), the less human capital a minority individual will accumulate in the
steady state.

Now, we add an additional consideration. Suppose when an individual accumulates
human capital up to a certain level, he can escape discrimination by becoming "highly"
skilled. For example, when an individual's superior human capital can situate him in a
position (or profession), not easily substituted by others, he will be certain to be employed
as a skilled worker regardless of his ethnic background.\(^35\)

Let $H^*$ denote the minimal amount of human capital that an individual has to accumu-
late in order to be "highly" skilled. Then, given his parental human capital $H_i$, the minimal
amount of effort that an individual has to spend on study, which is denoted by $e_{i+1}^m$, must
satisfy,

$$H^* = h(H_i, e_{i+1}^m)$$

(16)

Considering the maximal amount of effort that an individual can spend in study is one, we

\(^35\) A more realistic assumption might be that the probability that a minority individual will get a skilled
job increases as his human capital increases. But it is intuitive and not difficult to verify that we can get
qualitatively the same results based on either of the two assumptions. Thus, we might as well use the
assumption in the text for the maximal technical simplicity.
define $H^m$, which satisfies,

$$H^* = h(H^m, 1)$$  \(17\)

Since $h(H, e)$ is a strictly increasing function of both of its variables, it is impossible for an individual whose parental human capital is less than $H^m$ to accumulate enough human capital to be "highly" skilled. Thus, in what follows, we only need to discuss those individuals whose parental human capital is greater than $H^m$.

By the Implicit Function Theorem, $e^m_{t+1}$ is a function of $H_t$ (from (16)). We define it as,

$$e^m_{t+1} = e^m(H_t)$$

Thus, if an individual chooses to be "highly" skilled, his constraint is,

$$e_{t+1} \geq e^m(H_t)$$  \(18\)

Since an individual is certain that he will be employed as a skilled worker in this case, his intertemporal utility function is (7). So the Kuhn-Tucker condition is,

$$w_xu'(w_xh(H_t, e_{t+1}))h_2(H_t, e_{t+1}) - v'(e_{t+1}) \leq 0$$  \(19\)

$$(e_{t+1} - e^m(H_t))[w_xu'(w_xh(H_t, e_{t+1}))h_2(H_t, e_{t+1}) - v'(e_{t+1})] = 0$$  \(20\)

Similar to Lemma 3, we can show that the solution to (19) and (20) exists, and is unique. We denote it by $e^*_t$. Since it is a function of $H_t$, we define it as

$$e^*_{t+1} = e^*(H_t)$$

Now, we add an assumption that will serve as a sufficient condition of the following theorem.

**Assumption 6** (1) $\lim_{e \to 1} h(H^{\infty}, e) = H^*$; (2) there exists a large number $M$, such that $h(M, 0) = H^*$.

**Remark:** Under the above new consideration, a minority individual faces three occupational choices: being unskilled, trying to be skilled (with uncertainty of being successful), and
being “highly” skilled. The first part of the assumption emphasizes the difficulty of being "highly skilled" when an individual’s parental human capital is not very high, which also guarantees that these three choices are all effective options for individuals with different parental human capital.

Finally, we assume that $p$ is strictly less than one, namely, there is discrimination in the labor markets. Then we have the following theorem.

**Theorem 8** There exists a unique $H^\text{ccc}$ ($H^{\text{ccc}} > H^{\text{cc}}$), such that an individual will choose to be highly skilled if and only if his parental human capital is greater or equal to $H^{\text{ccc}}$.

**Remark.** This theorem indicates that an individual with $H_t$ amount of parental human capital will choose to be “highly” skilled if $H_t \geq H^{\text{ccc}}$; try to be skilled if $H^{\text{cc}} \leq H_t < H^{\text{ccc}}$; be unskilled if $H_t < H^{\text{cc}}$.

Based on the above theorem, we have the following corollary,

**Corollary 3** If $H^* > H^{\text{ccc}}$, then all of the offspring of a “highly” skilled worker will choose to be “highly” skilled.

This corollary indicates that some minority dynasties may separated themselves from others into a group of “highly” skilled dynasties in the long run. Furthermore, we have the following theorem,

**Theorem 9** If $H^* > \max(H^{\text{ccc}}, H^*)$, and if (12) is satisfied, then $H^*$ is the unique and stable steady state of individuals’ human capital of the group of “highly” skilled dynasties.

**Remark.** This theorem, together with Theorem 8 and Corollary 3, indicates that although some minority groups may become poor in a society due to the two adverse effects of discrimination (Theorem 5, 6, 7), some minority groups may become the upper income class of the economy as a result of trying to escape discrimination. Therefore, we have shown that discrimination in the labor markets pushes minority to the two extremes of an economy’s income distribution in the long run. Consequently, an interesting testable prediction of this study is that rich minority individuals’ income will decrease while poor minority individuals’ income may increase as discrimination against minorities is reduced.
Finally, the complicated dynamics of the evolution of minority individuals' human capital is illustrated in Figure 3,

Figure 3 is about here

which, in fact, also summarizes the basic results of this section.

6 Conclusion

Emphasizing the roles of an individual's effort in study and his parental human capital in his human capital formation, this paper demonstrates the possibility of persistent income inequality even if education is completely free. The model shows that when education is free (e.g. completely public), it is the initial parental human capital distribution that determines individuals' human capital formation and their occupational choice, and hence an economy's long-run income distribution. The study also examines the long run effects of some social factors on individuals' human capital formation. In particular, it indicates that as a result of labor market discrimination, minorities are pushed to the two extremes of an economy's income distribution, depending on the level of their initial human capital.

This study also provides an analytically tractable overlapping generations framework for a wide range of dynamic economic phenomena that emphasize formal schooling as the engine of growth, particularly when education is free (e.g. public). As illustrated in Section 5, there might be numerous the potential extensions and applications of this framework. For example: (a) the investigation of the permanent effect of some transitory productivity shock in the modern (skilled) sector on individuals' investment in human capital and hence on the long run output of an economy (see Fan and Spagat, 1994); (b) the examination of the effect of some random factors, such as "natural" learning ability or the uncertainty in the labor markets, on individuals' human capital formation and hence on intergenerational earnings mobility; and (c) The analysis of the dynamics of the evolution of individuals' human capital of an economy through the examination of the interaction (or combined effect) of (children's') formal schooling and (parents' or workers') learning by doing in an overlapping generations framework.
Finally, this model has an interesting empirical implication. The theory developed in this paper predicts that there will be (at least) two different income groups in the long run. Income converges within, but not across, different income groups. It also shows that individuals' occupational choices are endogenously determined by the level of their parental human capital. So, in contrast to the OLS (Ordinary Least Squares) methods employed in most relevant empirical studies, this model indicates that econometrically it is a self-selection problem and there is a non-linear relationship between children's earnings and parents' earnings. This novel result suggests a very promising future research avenue concerning the estimation of the correlation of intergenerational earnings.
References


Appendix: Mathematical Proofs

Proof of Lemma 3: Because $U$ is a continuous function with respect to $e_{t+1}$, and because $e_{t+1} \in [0, 1]$, which is a compact set, the optimal solution must exist. And because,
\[
\frac{d^2 U}{de_{t+1}^2} = w_s u'' h_2^2 + w_s u' h_{22} - v'' < 0
\]
the optimal solution must be unique.

Proof of Theorem 1: Totally differentiating (1) and rearranging, we get,
\[
\frac{dH_{t+1}}{dH_t} = h_1 + h_2 \frac{de_{t+1}}{dH_t}
\]  
(21)

Plugging (9) into (21), we get,
\[
\frac{dH_{t+1}}{dH_t} = \frac{h_1 v'' - w_s u' h_3 h_{22} + w_s u' h_{12}}{v'' - w_s^2 u'' h_2^2 - w_s u' h_{22}}
\]  
(22)

It is easy to see that both the numerator and the denominator of (22) are positive. Thus,
\[
\frac{dH_{t+1}}{dH_t} > 0
\]

Proof of Lemma 4: By the Envelope Theorem,
\[
\frac{\partial G}{\partial H_t} = u'(w_s H_{t+1}) w_s h_1 > 0
\]

And,
\[
\frac{\partial G}{\partial w_s} = H_{t+1} u'(w_s H_{t+1}) > 0, \quad \frac{\partial G}{\partial w_n} = -u'(w_n) < 0
\]

Proof of Theorem 2: Noticing assumption 3, we have,
\[
G(H_t = 0) = u(w_s h(0, e_{t+1}^*)) - v(e_{t+1}^*) - u(w_n) \\
\leq u(w_s h(0, e_{t+1}^*)) - u(w_n) \\
\leq u(w_s h(0, 1)) - u(w_n) \\
< u(w_s \frac{w_n}{w_s}) - u(w_n) \\
= 0
\]

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and
\[
G(H_t = N) = u[w_t h(N, e(N))] - v(e(N)) - u(w_n) \\
\geq u(w_t h(N, 0)) - v(0) - u(w_n) \\
= u(w_t h(N, 0)) - 0 - u(w_n) \\
> u(w_t \frac{w_n}{w_s}) - u(w_n) \\
= 0
\]

Thus, by the continuity of \(G()\), there must be a \(H^c\), such that
\[
G(H_t = H^c) = 0
\]

Besides, By Lemma 4, we know \(\frac{\partial G}{\partial H_t} > 0\). So \(H^c\) is unique.

Finally, noticing that
\[
u(w, h(H^c, e(H^c))) - v(e(H^c)) = u(w_n)
\] (23)

Totally differentiating (23) w.r.t. \(w_n\) and \(H^c\) and rearranging, we get,
\[
\frac{dH^c}{dw_n} = \frac{u'(w_n)}{w_n u'(w_t h(H^c, e(H^c))) h_1(H^c, e(H^c))} > 0
\]

Totally differentiating (23) w.r.t. \(w_s\) and \(H^c\) and rearranging, we get,
\[
\frac{dH^c}{dw_s} = -\frac{h(H^c, e(H^c))}{w_s h_1(H^c, e(H^c))} < 0
\]

**Proof of Theorem 3:** By Lemma 2, we see that when an individual chooses to be unskilled, the amount of time that he will spend on study is zero. Then, from Assumption 4, \(H = 0\) (and hence \(e = 0\)) is obviously the unique steady state. And at \(H = 0\), by Assumption 4,
\[
\frac{dH_{t+1}}{dH_t} = \lim_{H \to 0} \frac{h(H, 0) - 0}{H - 0} < 1
\]
So it is stable.

**Proof of Theorem 4:** (1) The proof of the first result is best illustrated in Figure 4,
where we can see that at $H_t = H^c$, by Assumption 5, we have $H_{t+1} > H_t$. Namely, the point of the locus of the dynamics of individuals' human capital at $H_t = H^c$ is above the 45 degree line.

Meanwhile, the concavity and the Inada condition of $h()$ can guarantee that there exist some $H^0$ (which is sufficiently large), such that

$$H^0 > h(H^0, 1)$$

Thus, when $H_t = H^0$, we have

$$H_{t+1} = h(H^0, e(H^0)) \leq h(H^0, 1) < H^0$$

Therefore, the point of the locus of the dynamics of individuals' human capital at $H_t = H^c$ is below the 45 degree line. Thus, this locus must cross the 45 degree line between $H_t = H^c$ and $H_t = H^0$ at least once. So we must have at least one steady state.

Also, Theorem 1 indicates that $\frac{dH_{t+1}}{dH_t} > 0$, namely, the locus of the dynamics of individuals' human capital is strictly upward sloping. Thus, at the first intersection between this locus and the 45 degree line, we must have $\frac{dH_{t+1}}{dH_t} < 1$, since this locus must cross the 45 degree line from above at this point (see Figure 4). Thus, this steady state is stable.

(2) To prove the second and the third result, we only need to show that when (12) is satisfied, we will have $\frac{dH_{t+1}}{dH_t} < 1$, $\forall H_t$. Noticing that $\frac{dH_{t+1}}{dH_t} > 0$ (Theorem 1), we see that from (22), $\frac{dH_{t+1}}{dH_t} < 1$ is equivalent to

$$h_1 v'' - w_s u' h_1 h_{22} + w_s u' h_2 h_{12} < v'' - w_s u'' h_2^2 - w_s u' h_{22}$$

Rearranging it, we get,

$$(1 - h_1) v'' - w_s u'' h_2^2 - w_s u' [ (1 - h_1) h_{22} + h_2 h_{12} ] > 0 \quad (24)$$

So when (12) is satisfied, (24) will be satisfied. Then,

$$0 < \frac{dH_{t+1}}{dH_t} < 1 \quad (25)$$
Thus, the steady state is globally stable, which automatically indicates it is unique.

**Proof of Corollary 2:** We will prove this corollary geometrically. See Figure 5:

Figure 5 is about here

if \( H_t < H^c \), similar to the situation in Theorem 3 or Corollary 1, the dynamics of individuals' human capital of this dynasty will reach the steady state \( H = 0 \).

On the other hand, if \( H_t \geq H^c \), because \( \left| \frac{dH_t+1}{dH_t} \right| < 1 \) when (12) is satisfied (by Theorem 4), the locus of the dynamics of individuals' human capital of a dynasty will never go through the 45 degree line; otherwise, we would have \( \left| \frac{dH_t+1}{dH_t} \right| \geq 1 \) (See figure 5). Thus, the steady state above \( H^c \) cannot exist. So the only steady state in this economy would be at \( H = 0 \).

**Proof of Theorem 5:** Firstly, we define,

\[
F \equiv p\left( w, h(H_t, e_{t+1}^p) \right) + (1 - p)u(w_n) - v(e_{t+1}^p) - u(w_n) \\
= p\left[ u(w, h(H_t, e^p)) \right] - u(w_n) - v(e_{t+1}^p)
\]

Clearly, \( F \) measures the difference between a minority individual's intertemporal utility of being skilled and being unskilled.

Noticing that \( F(H^c) = 0 \), namely,

\[
p\left[ u(w, h(H^c, e^p(H^c))) \right] - u(w_n) = v(e^p(H^c)) \tag{26}
\]

Totally differentiating (26) w.r.t. \( H^c \) and \( p \), using the Envelope Theorem, and rearranging, then we get,

\[
\frac{dH^c}{dp} = -\frac{u\left[ w, h(H^c, e^p(H^c)) \right] - u(w_n)}{pw_u[w, h(H^c, e^p(H^c))]h_1(H^c, e^p(H^c))} < 0
\]

**Proof of Theorem 6:** Totally differentiating (15) w.r.t. \( e_{t+1} \) and \( p \), and rearranging, we get,

\[
\frac{de_{t+1}}{dp} = \frac{w, u'h_2}{v'' - pw_u'u'h_2 - pw_u'u'h_{22}} > 0
\]

Q.E.D.
Proof of Theorem 7: By definition, the steady state of minority individuals' human capital must satisfy,

\[ pw_s u'(w_s H^d) h_2(H^d, e) = v'(e) \]  
\[ H^d = h(H^d, e) \]

(27)  
(28)

Totally differentiating (28) w.r.t. \( H^d \) and \( e \), we get

\[ \frac{de}{dH^d} = \frac{1 - h_1}{h_2} \]  
(29)

Then, totally differentiating (27) w.r.t. \( H^d \) and \( p \), and plugging (29) in, we get

\[ \frac{dH^d}{dp} = \frac{w_s u'h_2}{v'(\frac{1 - h_1}{h_2} - ph_2 u'' - pw_s u' h_3 h_2 + (1 - h_1) h_2)} \]  
(30)

When (12) is satisfied, both the numerator and the denominator of (30) are positive (it should be noted that (12) entails \( h_1 < 1 \)), so

\[ \frac{dH^d}{dp} > 0 \]

Q.E.D.

In order to prove Theorem 8, we firstly try to establish two lemmas.

Lemma 5 For any \( H_t \), when \( p < 1 \),

\[ e^s(H_t) > e^p(H_t) \]

Proof of Lemma 5: Suppose not, namely suppose \( e^s(H_t) \leq e^p(H_t) \). Then

\[ v'(e^s(H_t)) \leq v'(e^p(H_t)) \]

From (15), we have,

\[ pw_s u'(w_s h(H_t, e^p(H_t))) h_2(H_t, e^p(H_t)) = v'(e^p(H_t)) \]

(31)

From (19), we have,

\[ w_s u'(w_s h(H_t, e^p(H_t))) h_2(H_t, e^s(H_t)) \leq v'(e^s(H_t)) \]

(32)

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Therefore, we have,

$$pw_s u'(w_s h(H_t, e^p(H_t))) h_2(H_t, e^p(H_t)) \geq w_s u'(w_s h(H_t, e^*(H_t))) h_2(H_t, e^*(H_t))$$  \hspace{1cm} (33)

However, because

$$\frac{d}{de} [w_s u'(w_s h(H_t, e)) h_2(H_t, e)] = w_s^2 u'' h_2^2 + w_s u' h_{22} < 0$$

$e^*(H_t) \leq e^p(H_t)$ implies

$$w_s u'(w_s h(H_t, e^p(H_t))) h_2(H_t, e^p(H_t)) \leq w_s u'(w_s h(H_t, e^*(H_t))) h_2(H_t, e^*(H_t))$$  \hspace{1cm} (34)

Thus, when $p < 1$,

$$pw_s u'(w_s h(H_t, e^p(H_t))) h_2(H_t, e^p(H_t)) < w_s u'(w_s h(H_t, e^*(H_t))) h_2(H_t, e^*(H_t))$$  \hspace{1cm} (35)

This leads to contradiction to (33). Thus, $e^*(H_t) > e^p(H_t)$.

**Lemma 6**

$$\frac{dU}{dH_t} = \frac{h_1(H_t, e^t_{t+1})}{h_2(H_t, e^t_{t+1})} v'(e^t_{t+1})$$  \hspace{1cm} (36)

$$\frac{dV}{dH_t} = \frac{h_1(H_t, e^p_{t+1})}{h_2(H_t, e^p_{t+1})} v'(e^p_{t+1})$$  \hspace{1cm} (37)

**Proof of Lemma 3:** (1) When the Kuhn-Tucker condition of (19) holds with strict equality, by the Envelop theorem,

$$\frac{dU}{dH_t} = w_s h_1 u'(w_s H^*_{t+1}) = \frac{h_1}{h_2} v'(e^*_{t+1})$$

(2) When the Kuhn-Tucker condition of (19) holds with strict inequality, $e^t_{t+1}$ is a corner solution, by continuity, (19) holds with strict inequality when $H_t$ changes a little bit. Namely, $e^*_{t+1}$ remains a corner solution. Therefore, by $H^* = h(H_t, e^*_{t+1})$, we have,

$$\frac{de^*_{t+1}}{dH_t} = -\frac{h_1}{h_2}$$  \hspace{1cm} (38)
\[
\frac{dU}{dH_t} = w_s h_1 u'(w_s H_{t+1}^e) + \frac{dU}{de_{t+1}^e} \frac{de_{t+1}^e}{dH_t} \\
= w_s h_1 u' + (w_s h_2 u' - v'(-\frac{h_1}{h_2})) \\
= \frac{h_1}{h_2} v' 
\]

Thus, in either case, (36) holds.

The proof of the second part of the Lemma is similar to part (1) of the above proof.

Proof of Theorem 8: Firstly, we define

\[
X(H_t) \equiv U(e^s(H_t)) - V(e^p(H_t)) \\
= u(w_s h(H_t, e^s(H_t))) - v(e^s(H_t)) - [pu(w_s h(H_t, e^p(H_t))) + (1-p)u(w_n) - v(e^p(H_t))] 
\]

Clearly, \(X(H_t)\) measures the difference of a minority individual’s intertemporal utility between being “highly” skilled and trying to be skilled. From Assumption 6, similar to the proof of Theorem 2, it is easy to see \(X(H^{cc}) < 0\) and \(X(M) > 0\). Thus, there exists \(H^{cc}\) \((H^{cc} < H^{ccc})\), such that \(X(H^{ccc}) = 0\). Now we try to prove \(\frac{dX}{dH_t} > 0\).

By Lemma 6, we have,

\[
\frac{dX}{dH_t} = \frac{dU}{dH_t} - \frac{dV}{dH_t} \\
= \frac{h_1(H_t, e_{t+1}^s)}{h_2(H_t, e_{t+1}^s)} v'(e_{t+1}^s) - \frac{h_1(H_t, e_{t+1}^p)}{h_2(H_t, e_{t+1}^p)} v'(e_{t+1}^p) 
\]

Because,

\[
\frac{d[h_1(H_t, e)/h_2(H_t, e)]}{de} = \frac{h_{12} h_2 - h_{22} h_1}{h_2^2} > 0 
\]

And because \(e^s(H_t) > e^p(H_t)\) (by Lemma 5)

\[
\frac{h_1(H_t, e^s(H_t))}{h_2(H_t, e^s(H_t))} > \frac{h_1(H_t, e^p(H_t))}{h_2(H_t, e^p(H_t))}, \quad v'(e^s(H_t)) > v'(e^p(H_t)) 
\]

Therefore,

\[
\frac{dX}{dH_t} > 0 
\]
So, when $H_t \geq H^{cc}$, $X(H_t) \geq 0$. Thus, in this case, a minority individual will choose to be “highly” skilled over trying to be skilled, which is a better choice over being unskilled (because $H_t \geq H^{cc} > H^{cc}$). Therefore, we have proved the theorem.

**Proof of Corollary 3**: Similar to the first part of the proof of Corollary 1.

**Proof of Theorem 8**: Let $H'$ denote the steady state(s) of individual’s human capital of “highly” skilled dynasties. Then, it satisfies

$$K(H', a(H')) = w_s u'[w_s h(H', a(H'))]h_2(H', a(H')) - v'(a(H')) \leq 0 \quad (41)$$

Because when (12) is satisfied,

$$\frac{dK}{dH'} = w_s^2 u'' h_2 - v'' \frac{de}{dH'} + w_s u'[h_{12} + h_{22} \frac{de}{dH'}]$$

$$= w_s^2 u'' h_2 - v'' \frac{1 - h_1}{h_2} + w_s u' \frac{(1 - h_1) h_{22} + h_2 h_{12}}{h_2}$$

$$< 0 \quad (42)$$

Also, by definition, $K(H^*, a(H^*)) = 0$. So, when $H' \geq H^* > H^*$, we have,

$$K(H', a(H')) < K(H^*, a(H^*)) = 0$$

Thus, $H'$ must be the corner solution, namely, $H' = H^*$.

Meanwhile, by continuity, an individual will still choose the corner solution (namely, (19) still holds with strict inequality) when his parental human capital deviates a little from $H^*$, so at $H^*$, \( \frac{dH_{i+1}}{dH_i} = 0 \). So it is stable.
Figure 1: 
\[ y = \max\{w, w H\} \]

\[ y \]— An individual's income
Figure 2(b): When multiple equilibria exist within the skilled group
Figure 3: the locus of the dynamics of minority individuals' human capital
Figure 5: The only steady state is $H=0$