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Testing for a Nonlinear Relationship among Fundamentals and Exchange Rates in the ERM

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Abstract

We employ two nonparametric nonlinear testing methodologies, namely a nonparametric nonlinear cointegration approach and a nonlinear Granger causality approach, to test for a nonlinear relationship between macroeconomic fundamentals and exchange rates for two country-pairs, namely the Netherlands-Germany and France-Germany. The results suggest that there is nonlinear cointegration among money, output and exchange rates for Netherlands-Germany, which can be interpreted as evidence of a long-run nonlinear relationship. For France-Germany, we fail to find evidence of nonlinear cointegration, but we find nonlinear Granger causality from French money to the FFr/DM exchange rate. These findings may be interpreted as evidence of a dynamic nonlinear relationship and are consistent with the German dominance hypothesis. On the basis of estimated fractional ARIMA models, we rejected the hypothesis that these nonlinearities are due to bubbles.

Keywords: Nonlinearity, Cointegration, Causality, Exchange rates.


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I. Introduction

Since the seminal work of Meese and Rogoff (1983) which documented the failure of some linear exchange rate models, several more recent studies (Flood and Rose, 1995; Rose, 1996) have provided further evidence of the empirical failure of the linear models. The theoretical extension of the linear exchange rate framework to nonlinear behaviour has been growing in the literature. These nonlinear extensions include the concept of bubbles with self-fulfilling expectations (see Flood and Garber, 1980; Blanchard and Watson, 1982; Froot and Obstfeld, 1991), target zone models (Krugman, 1991), a stop-loss trading strategy of traders (Krugman and Miller, 1993), nonlinear monetary strategies (Flood and Isard, 1989), and fads or noise trading (see, for example, Shiller 1984; Kyle, 1985; Black, 1986; Frankel and Froot, 1986; Summers, 1986; and De Long et al, 1990). Empirical studies have mainly tested for nonlinearities due to target zones, and have failed to support such nonlinearities (Meese and Rose, 1990; and Flood et al, 1991; Lindberg and Soderlind, 1994). However, it remains an open question whether there exist general type nonlinearities which are consistent with the other nonlinear models.

In this paper, we propose two nonparametric tests for the nonlinearities in the exchange rate-fundamentals relationship. The first is a nonlinear cointegration test (Granger and Hallman, 1991; Breiman and Friedman, 1985), which we regard as a test for a nonlinear long-run relationship between exchange rates and fundamentals. The second test is a nonlinear Granger causality test (Baek and Brock, 1992; Hiemstra and Jones, 1994), which is regarded as a test for a nonlinear dynamic relationship. This test can detect the dynamic nonlinear Granger-causal relationship between exchange rate and fundamentals by testing whether the past values of the fundamentals influence
present and future of the exchange rates. Evidence of nonlinear causality can be interpreted as evidence that the underlying dynamic relationship between exchange rates and fundamentals is nonlinear. It is beyond the scope of this paper to identify to which particular theoretical model the revealed nonlinearity can be attributed. However, the particular type of nonlinearities due to bubbles can be tested for using the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model (see Granger and Joyeux, 1980, Geweke and Porter-Hudak, 1983) which, according to Scacciavillani (1994), provides a promising empirical test against intrinsic bubbles¹.

Our findings support the nonlinearity of exchange rate models based on a simple reduced-form monetary model. After establishing that there are no bubbles, we find evidence of nonlinear cointegration between the Dutch Guilder - Deutsche Mark (GUI/DM) exchange rate and its fundamentals of money and output. This finding can be interpreted as evidence that there is a long-run nonlinear relationship between the GUI/DM and its fundamental variables. For the case of the French Franc - Deutsche Mark (FFr/DM) exchange rate, we find evidence of no nonlinear cointegration but dynamic nonlinear Granger-causality from French money to the FFr/DM exchange rate. There is no evidence of Granger-nonlinear causality from German money to the FFr/DM rate. These results may be interpreted as evidence to support a dynamic nonlinear relationship for this exchange rate, which is also consistent with the German dominance hypothesis (see Artis and Nachane, 1990, and De Grauwe, 1994). As discussed above, these nonlinearities cannot be attributed to bubbles or to a specific theoretical model such as the stop-loss model. However, these results carry some

¹ Earlier empirical work have focused on testing for `intrinsic` bubbles based on integer order of integration tests. These tests, nevertheless, have low power to detect for `intrinsic` bubbles (Evans,
implications regarding the failure of previous studies in detecting specific target zone nonlinearities: the existence of general type nonlinearities may distort possible target zone specific, S-shaped, nonlinearities rendering the previous studies unable to trace these specific nonlinearities.

The remainder of the paper is organised as follows. Section II gives a brief introduction to some theories which indicate a nonlinear relationship between exchange rate and fundamentals. Section III introduces a nonlinear testing procedure which consists of the nonlinear nonparametric cointegration technique and the modified Back and Brock's test for dynamic nonlinear Granger causality. Section IV describes the data used and Section V presents the results. Finally, Section VI concludes, with suggestions for a useful direction for future research in this area.

II. Nonlinearities in the relationship between exchange rates and fundamentals: Theories and empirical evidence.

The theoretical models which, explicitly or implicitly, establish a nonlinear exchange rate-fundamentals relationship can be categorised as bubbles models, target zone models, models of microfoundation of trading behaviour, models of government monetary policies, and models of fads or noise trading.

Flood and Garber (1980) introduced the concept of bubbles with self-fulfilling expectations. Blanchard and Watson (1982) presented a special version of the bubble as rational bubbles. Rational bubbles are generated by extraneous events or rumours and driven by self-fulfilling expectations which has nothing to do with the

fundamentals. Froot and Obstfeld (1991) introduced intrinsic bubbles which, unlike the rational bubbles, are driven by fundamentals alone in a nonlinear way. Ikeda and Shibata (1995) develop the intrinsic bubble into exchange rate, thereby entailing a nonlinear relationship between the exchange rate and fundamentals.

Krugman (1991) introduced the target zone model and derived a specific, S-shaped, nonlinear relationship between exchange rate and fundamentals. This is due to the forward looking assumption of the exchange rate. As the exchange rate moves close to the band, forward-looking agents anticipate an intervention and incorporate this into their expectations. Repeated revisions of exchange rate expectations will lead to an S-shaped relationship between exchange rates and fundamentals.

Krugman and Miller (1993) derived a nonlinear relationship between exchange rates and fundamentals within a free-floating setting, based on the micro-foundation of trading behaviour. The nonlinearity now arises due to a change in the risk premium caused by stop-loss trading strategies adopted by speculative foreign investors. These investors are assumed to hold domestic-currency denominated assets, and wish to protect themselves against decreases in the value of the domestic currency by selling their domestic assets (‘exiting’ the market) if the exchange rate falls below a trigger value. Exiting the market entails a change in the risk premium on the domestic assets. Moreover, when the stop-loss traders exit the market, the other (non stop-loss) traders buy domestic assets and sell foreign assets. This in turn causes a change in the risk premium on the foreign assets. This risk premium change entails a break in the free-

\footnote{The target zone model effectively assumes a fixed risk premium at any level of the exchange rate.}
floating exchange rate path, thereby establishing a nonlinear (piecewise linear) relationship between the exchange rate and the fundamental.

Fads or noise trading may also create persistent departures from the linear relationship of the exchange rate and fundamentals (see, for example, Shiller 1984; Kyle, 1985; Black, 1986; Frankel and Froot, 1986; Summers, 1986; and De Long et al, 1990). In a fads model, investors perceive psychological barriers to upward movements in the exchange rate. When a trigger exchange rate is hit, noise traders enter (or exit) the market and consequently, a new trigger exchange rate is established. Nonlinearity is therefore created by the speculative behaviour of rational investors.

Nonlinearity in government monetary policies is another factor to which a nonlinear exchange rate-fundamental relationship can be attributed. This strand of literature addresses the issue of speculative attacks on government-controlled exchange rates (see Flood and Marion (1998) for an excellent review). A first case of policy nonlinearity is a shift in the growth rate of domestic credit (a constituent of domestic money supply) conditional on whether there is an attack or not\(^3\). Each growth rate reflects a path that the exchange rate follows. When speculators attack and the growth rate of credit changes, the exchange rate jumps from the non-attack path to the attack path, thereby establishing a nonlinearity in the exchange rate-fundamental relationship. A second policy nonlinearity is when the government follows a simple linear rule with an escape clause\(^4\); the nonlinearity is now established in the face of self-fulfilling speculative attacks.

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\(^3\) This policy nonlinearity first appeared in Flood and Garber (1984).

\(^4\) Such a rule was introduced by Flood and Isard (1989).
Previous empirical studies testing for a nonlinear exchange rate-fundamentals relationship focused on specifically testing the S-shaped type of nonlinearity as suggested by the target zone model. Meese and Rose (1990, 1991) used nonparametric nonlinear tests and failed to find evidence of target zone specific nonlinearities for several US dollar exchange rates during the Bretton Woods period and also for ERM exchange rates during a more recent period. Similarly, Flood et al (1991) failed to find target-zone nonlinearities for ERM countries using parametric nonlinear tests on daily data for exchange rates and fundamentals, and Lindberg and Soderlind (1994) also rejected the target zone model using Swedish data.

In this paper, we seek to extend the findings of previous studies by utilising two recently developed nonparametric approaches in testing for nonlinearities in the exchange rate-fundamentals relationship. The first approach is a nonlinear nonparametric cointegration test, developed by Granger and Hallman (1991), and Breiman and Friedman (1985), and is used for testing a long-run nonlinear relationship between exchange rates and fundamentals. The second approach is a nonlinear Granger causality test, originated by Baek and Brock (1992) and modified by Hiemstra and Jones (1994), and is used for testing a dynamic nonlinear relationship. We also discuss how these two approaches could be combined in an integrated procedure for nonlinearity testing (see Figure 1 later in text).

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5 Lindberg and Soderlind (1994b) and Werner (1995) have found some evidence to support the mean-reverting intra-marginal interventions model for the Swedish krona and some ERM exchange rates on the basis of implications of uncovered interest parity under target zone. Moreover, Rose and Svensson (1991) and Bertola and Svensson (1993) applied univariate distribution tests to FFR/DM exchange rates and also find some evidence to support the imperfect credible target zone model.
III. A testing procedure for nonlinearity.

In this section, we present the two approaches for nonlinearity testing, namely the nonlinear cointegration approach and the nonlinear Granger causality approach. We then combine these two approaches in an integrated procedure, which carefully discusses how to implement these sophisticated tests and addresses issues related to their application.

III.A. The nonparametric nonlinear long-run relationship test

Breiman and Friedman (1985) provide a nonparametric approach to find nonlinear transformations on variables to minimise the expected mean squared error of the additive model:

\[ f(y) = g_1(x_1) + \ldots + g_k(x_k) + e. \]  

The idea is to estimate a series of 'alternating conditional expectations' (ACE) of the variables, using good estimates of \( f(.) \) and \( g_i(.) \) \( (i=1, \ldots, k) \). Breiman and Friedman show that the ACE algorithm provides transformations which asymptotically converge to the optimal transformations. A possible consequence of this algorithm is that it may cause an original I(1) series to become I(0) after transformation (Granger and Hallman, 1991).

These optimally and nonlinearly ACE-transformed variables may then be used for a linear cointegration test, provided that they maintain the nonstationarity property. If a linear cointegration is found, then it could be characterised as a finding of nonlinear
cointegration among the original un-transformed variables (Granger and Hallman, 1991, and Meese and Rose, 1991).

The detailed testing procedure involves two stages, as suggested by Granger and Hallman (1991). Firstly, the ACE-transformed variables are tested for whether the nonstationarity is maintained by using unit root tests with bootstrapped critical values. Secondly, linear parametric and nonparametric cointegration techniques are applied to the ACE-transformed series if they remain I(1) after transformation and the original I(1) series if they become I(0) after transformation. In this paper, we employ both parametric (Johansen, 1988) and nonparametric (Bierens, 1997a) cointegration procedures as they are complimentary with each other (Bierens, 1997a).

III.B. The nonparametric nonlinear dynamic relationship test.

Baek and Brock (1992) proposed a nonparametric statistical method for detecting nonlinear causal relations between two time series. This method was modified by Hiemstra and Jones (1994) to allow each series to display weak temporal dependence. To define nonlinear Granger causality, assume there are two strictly stationary and weakly dependent scalar time series \( \{W_i\} \) and \( \{V_i\} \). Define the \( m \)-length lead vector of \( W_i \) by \( W_i^m \), and the \( Lw \)-length and \( Lw \)-length lag vectors of \( W_i \) and \( V_i \), respectively, by \( W_i^{Lw} \) and \( V_i^{Lw} \):

\[
W_i^m = (W_i, W_{i+1}, ..., W_{i+m}), \quad m = 1, 2, ..., \quad t = 1, 2, ...
\]

\[
W_i^{Lw} = (W_{i:Lw}, W_{i:Lw+1}, ..., W_i), \quad Lw = 1, 2, ..., \quad t = Lw+1, Lw+2, ...
\]  

\[
V_i^{Lw} = (V_{i:Lw}, V_{i:Lw+1}, ..., V_i), \quad Lw = 1, 2, ..., \quad t = Lw+1, Lw+2, ...
\]

(2)

The definition of nonlinear Granger noncausality is given by equation (3):
\[ \Pr\{ \| W_{i}^m - W_{e}^m \| < e \mid \| W_{i-Lw}^{Lw} - W_{e-Lw}^{Lw} \| < e, \| V_{i-Lv}^{Lv} - V_{e-Lv}^{Lv} \| < e \} = \]
\[ = \Pr\{ \| W_{i}^m - W_{e}^m \| < e \mid \| W_{i-Lw}^{Lw} - W_{e-Lw}^{Lw} \| < e \} \] (3)

where \( \Pr\{ \} \) is probability and \( \| \cdot \| \) is the maximum norm. If equation (3) holds, for given values of \( m, Lw, \) and \( Lv \geq 1 \) and for \( e > 0 \), then \( \{ V_i \} \) does not strictly Granger cause \( \{ W_i \} \). This definition of nonlinear Granger causality is based on two conditional probabilities. The probability on the left hand side of equation (3) can be interpreted as the conditional probability that any two \( m \)-length lead vectors of \( \{ W_i \} \) are within a metric \( e \) of each other, given that the corresponding \( Lw \)-length lag vectors of \( \{ W_i \} \) and \( Lv \)-length lag vectors of \( \{ V_i \} \) are within a distance \( e \) of each other. The probability on the right hand side of equation (3) can be interpreted in a similar way to that on the left hand side except we now replace lead vectors by lag vectors. The null hypothesis is that \( \{ V_i \} \) does not nonlinearily Granger cause \( \{ W_i \} \). Under the null hypothesis, and for given values for \( m, Lw, Lv \) and \( e > 0 \), it can be shown that the statistic

\[ \sqrt{n} \left\{ \frac{C_1(m+Lw, L_v, e, n)}{C_2(m+Lw, e, n)} - \frac{C_3(m+Lw, e, n)}{C_4(Lw, e, n)} \right\} \sim \text{AN} (0, \sigma^2(m, Lw, Lv, e)) \] (4)

where \( C_1(m+Lw, L_v, e, n), C_2(m+Lw, e, n), C_3(m+Lw, e, n), \) and \( C_4(Lw, e, n) \) are correlation-integral estimators of the point probabilities corresponding to the left hand side and right hand side of equation (3). This test has very good power properties against a variety of nonlinear Granger causal and noncausal relations, and its asymptotic distribution is the same if the test is applied to the estimated residuals from a vector autoregressive (VAR) model (Hsieh and Jones, 1994).

III.C. A testing procedure for nonlinearity
Figure 1. A procedure for testing nonlinearity

Step 1. Test for Nonstationarity

$I(1)$ series

stationary series

Step 2. Test for linear cointegration

linear cointegration not found

linear cointegration found

Step 3. Test for nonlinear cointegration

nonlinear cointegration found

nonlinear cointegration not found

Step 4. Test for nonlinear Granger causality

nonlinear Granger causality found

nonlinear Granger causality not found

Conclusion: Evidence of long-run nonlinearity
Evidence of dynamic nonlinearity
No evidence of nonlinearity
As the issue of concern here is the nonlinear relationship beyond linearity, we need to remove the linear structure of the variables before applying the nonlinear tests. As suggested by Baek and Brock (1992) and Hiemstra and Jones (1994), we shall extract the residuals from a VAR model which contains the variables under consideration. A problem which arises immediately from this approach is that most of economic time series are nonstationary. Without proper treatment of nonstationarity and possible cointegration, the estimated VAR are subject to mis-specification problem. We therefore propose the following procedure (see Figure 1) which takes careful account of this issue. Our procedure is outlined in the following steps:

**Step (1)**: Unit root tests are employed to determine whether the time series are I(0) or I(1). If the series are nonstationary, then proceed to Step (2). Otherwise proceed to Step (4).

**Step (2)**: Test for linear parametric and nonparametric cointegration using Johansen’s (1988) and Bierens’ (1997a, 1997b) approaches. If cointegration is not found, proceed to Step (3). Otherwise, proceed to Step (4).

**Step (3)**: Given that there is no linear cointegration, test for nonlinear cointegration on the basis of the approach discussed in section II. If there exists nonlinear cointegration, it may be interpreted as strong evidence of long run relationship. Otherwise, proceed to Step (4).

**Step (4)**: Apply the modified Baek and Brock (1992) nonlinear Granger causality test to the residuals from a VAR. The purpose of applying the test to residuals from a VAR is to remove all linear structure. Moreover, as the test requires the residuals to be stationary, caution should be exercised as to whether the VAR should be constructed by variables in levels or in first differences. If the outcomes from

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6 For more details on the derivations, see Hiemstra and Jones (1994).
the preceding steps are that the series are stationary [Step (1)], or that linear cointegration is found [Step (2)], then a VAR based on the time series in levels should be estimated. If the outcome of the preceding step is that nonlinear cointegration is not found [Step (3)], a VAR based on time series in first differences should be estimated.

As discussed above, the finding of nonlinear causality implies a dynamic relationship between exchange rate and fundamentals. Finally, no causality implies that there is no evidence of nonlinearity. Finally, to rule out the possibility that any potential nonlinearity is due to bubbles, an ARFIMA model is applied to test the noninteger order of integration of all the time series. If the order of integration of the exchange rates is not significantly higher than that of the fundamentals, then the existence of bubbles is ruled out.

One limitation of our proposed testing procedure for nonlinearity is that we cannot proceed to test for nonlinear cointegration if there is linear cointegration. Therefore, we can only proceed to test for nonlinear Granger causality [the move from Step (2) to Step (4)].

**IV. Data**

To illustrate the implementation of the above testing procedure for nonlinearity, we focus on three countries in the ERM, namely the Netherlands, Germany and France, which form two pairs: the Netherlands-Germany and France-Germany. We focus on these three countries which are regarded as core members in the inception and operation of the ERM target zones whose existence has been rejected by empirical studies. This will provide the motivation to test the existence of other forms of
nonlinearity derived from the theory. It will also provide us a starting point for further research on all the ERM countries in the future.

Two bilateral ERM exchange rates are examined, namely the Dutch Guilder / Deutsche Mark (Guilder/DM), French Franc / Deutsche Mark (FF/DM). These are exchange rates between currencies which participated in the ERM since its inception. To measure fundamentals, two alternative variables are used, namely money supply and output of the countries concerned. This choice of fundamentals is based on the fact that money and output are the traditional reduced-form determinants of the exchange rate in the context of flexible-price monetary exchange rate models (Flood et al., 1991).

Given the fact that the quarterly data do not provide us sufficient number of observations and the daily data do not have information on the fundamentals of money and output, we choose monthly observations obtained from Datastream. The data span the period from January 1980 to October 1996, giving a total of 202 observations. Exchange rate data are end-of-the-month values. Money supplies are nominal M1, and output is approximated by industrial production at constant prices. The data are seasonally adjusted, and expressed in natural logarithms.

V. Results.

V.A. Testing for nonstationarity [Step (1) in Figure 1]

The results from testing for nonstationarity (step 1 in our testing procedure) are reported in Table 1. This Table reports the Augmented Dickey-Fuller (ADF) test for the levels and first differences of all the variables involved. All the variables appear to be nonstationary in levels and stationary in first differences, i.e. integrated of order 1
[I(1)]². As all the variables are I(1), we proceed to Step (2) to test for linear cointegration according to Figure 1.

V.B. Linear cointegration tests [Step (2) in Figure 1]

(i) The Netherlands - Germany

Tables 2 and Table 3 present, respectively, the Johansen maximum likelihood and the Bierens’ nonparametric estimation results of linear cointegration between the GUK/DM exchange rate, money supply and output. Both methods fail to find evidence of linear cointegration. So, for the case of the Netherlands-Germany, we will proceed to Step (3) to test for nonlinear cointegration.

(ii) France - Germany

At the 5% significance level, both Johansen’s and Bierens’ approaches suggest there are 2 cointegration vectors between the FFr/DM exchange rate and the money supply variables (Tables 2 and 3). Furthermore, a likelihood ratio test on exchange rate deletion from the two cointegration vectors rejected the null hypothesis that the exchange rate can be excluded from the two cointegration vectors at the 5% significant level. Therefore we conclude that there is linear cointegration between the FFr/DM exchange rates and its fundamentals. So, for the case of France-Germany, we will proceed to Step (4) to test for nonlinear causality.

² We have also implemented the KPSS unit root test proposed by Kwiatkowski et al (1992). Results are similar to those obtained from the ADF test, and are available on request.
V.C. Nonlinear cointegration tests for the Netherlands vs Germany case [Step (3) in Figure 1].

From Step (2), both Johansen’s and Bierens’ approaches found no evidence of linear cointegration for the Netherlands and Germany. In this sub-section, we investigate whether there may exist nonlinear cointegration between the GUI / DM exchange rate and its fundamentals.

In order to conduct the nonlinear cointegration analysis, we first transform the five variables of bilateral exchange rate, money supply and outputs of the Netherlands and Germany by the method of alternating conditional expectations (ACE). We next need to confirm that the ACE-transformed variables are I(1). To this aim, we apply the ADF unit root test to the transformed variables and construct tailor-made critical values using bootstrap simulations. As shown in Table 4, the two transformed output variables remained as I(1) whilst the other three transformed variables, i.e. GUI/DM exchange rate and two money variables become I(0) after the ACE transformation. To conduct cointegration analysis among these five variables, we therefore used the original (log) bilateral exchange rate and money supplies, which have been established as I(1) in Table 1, and the ACE transformed outputs, which are also I(1).

Table 5 reports the results from the nonlinear cointegration tests. Johansen’s parametric method failed to find any of the cointegration vector between the GUI/DM, and the original money supplies and ACE transformed outputs of the two countries. However, the Bierens’ nonparametric cointegration approach finds three cointegration vectors for these five variables. Furthermore, an exchange rate deletion test suggests that the exchange rate is significant in these three cointegration vectors. Therefore, the
Bierens’ nonparametric approach suggests there is nonlinear cointegration for the Netherlands-Germany case, whilst the Johansen’s parametric approach failed to find any evidence of cointegration. The existence of nonlinear cointegration for the Netherlands-Germany case can be interpreted as evidence of a long-run nonlinear relationship between the GUI/DM rate and money and output of the two countries.

V.D Nonlinear Granger causality test for France-Germany [Step (4) in Figure 1].
As we found [Step (2)] that there is linear cointegration between the FFr/DM exchange rates and the money supplies of the two economies, we proceed to test for nonlinear causality from money to exchange rates according to our testing procedure.

To test for nonlinear Granger causality from money supplies to exchange rate, we initially estimate the following VAR:

\[
\begin{align*}
    s_t &= A(L) s_t + B(L) M_{G,t} + C(L) M_{F,t} + U_{s,t} \\
    M_{G,t} &= C(L) s_t + D(L) M_{G,t} + E(L) M_{F,t} + U_{G,t} \\
    M_{F,t} &= H(L) s_t + K(L) M_{G,t} + N(L) M_{F,t} + U_{F,t}
\end{align*}
\]  

where \(s_t, M_{G,t}\), and \(M_{F,t}\) are (the logarithm of) the exchange rate and money supplies of Germany and France, respectively. The optimal lag length for each VAR is determined using likelihood ratio tests. On the basis of the chosen lag length\(^8\), we estimate (6) using OLS, and obtain the estimated residuals from (5), \(\hat{U}_{s,t}\) and \(\hat{U}_{G,t}\) and \(\hat{U}_{F,t}\). The stationarity of these three residuals series is guaranteed by the existence of cointegration relation among the three variables in the VAR.

\(^8\) We started with an upper limit of 18 lags, and sequentially tested for 15, 12, 9, 6, 3, and 1 lags using likelihood ratio tests. On the basis of these tests, a lag length of 15 lags is chosen for the VAR of equation (5).
To test for nonlinear Granger causality from French and German money to the FFr/DM exchange rate, the modified Back and Brock statistic is applied to the estimated residuals from (6), \( \{ \hat{U}_{st}, \hat{U}_{gt}\} \) and \( \{ \hat{U}_{st}, \hat{U}_{rt}\} \). Before conducting the tests, values for the lead length \( m \), the lag lengths \( L_w \) and \( L_v \), and the scale parameter \( e \) must be chosen [see eq.(3) and (4)]. On the basis of the Monte Carlo results of Hiemstra and Jones (1993), we set for all cases, \( m=1, L_w = L_v \) using common lag lengths of 1 to 12 lags. Moreover, for all cases, we set \( e = 1.6\sigma \), where \( \sigma = 1 \) denotes the standard deviation of the standardised time series.

Table 6 reports the results of nonlinear Granger causality from money supplies to FFr/DM exchange rate. The null hypothesis that French money supply does not nonlinearly Granger cause FFr/DM exchange rate is rejected at the 5% level of the significance. The results are consistent for lags from 8 up to 12 months. We do not, nevertheless, find any evidence of nonlinear causality from German money to FFr/DM exchange rate. This result shows that there is evidence of a 'dynamic' nonlinear relationship of the bilateral rate between France and Germany. The FFr/DM exchange rate is influenced by French money supply dynamically rather than by German money. This is consistent with the German dominance hypothesis that the interventions on foreign exchange market from German side are more outward looking oriented whilst the French interventions are more concerned with the FFr against DM (see, Artis and Nachane, 1990, De Grauwe, 1994)

In conclusion, for the case of the Netherlands-Germany, we have gone through Steps (1), (2), and (3) in Figure 4, and found strong evidence to support a long-run nonlinear relationship. For the case of France-Germany, we have implemented Steps (1), (2), and
(4), and found evidence of a 'dynamic' nonlinear relationship between their bilateral rate and fundamentals. To rule out the possibility that these nonlinearities are due to bubbles, we implement bubbles tests based on estimating the noninteger order of integration of the exchange rates and the fundamental variables using ARFIMA models. If the order of integration of the exchange rates is the same as the order of integration of fundamentals then we can rule out the existence of bubbles (see Scacciavillani, 1994). Table 7 reports the results. These results suggest that the order of integration of each of the (log of) money supply, output and exchange rate series is not significantly different from 1 at the 5% level, according to the asymptotic normal statistics. Since the exchange rates are also integrated of order 1, we conclude that there are no bubbles in the two exchange rates of Guilder/DM and FFr/DM and therefore, any nonlinearities can be attributed to bubbles.

VI. Conclusion

This paper employs two nonparametric testing approaches to examine whether there is a nonlinear relationship between fundamentals and exchange rates for three countries, Germany, the Netherlands and France. The first approach is a nonlinear cointegration test aimed at examining the existence of a long-run nonlinear relationship, and the second is a nonlinear Granger-causality test aimed at revealing a dynamic nonlinear relationship. Two alternative measures of fundamentals were adopted: money supply and output. We find that there exists nonlinear cointegration between the exchange rate and money supply for the Netherlands-Germany case, which can be interpreted as evidence of a long-run nonlinear relationship of the fundamentals and the bilateral rate for these countries. For the France-Germany case, we find evidence of nonlinear Granger causality from French money to the FFr/DM exchange rate, thereby
establishing that there exists a dynamic nonlinear relationship between fundamentals and the FFr/DM exchange rate. This finding is also consistent with the German dominance hypothesis in the ERM (Artis and Nachane, 1990). We confirmed, using ARFIMA models, that these nonlinearities are not due to bubbles. These results have an important implication which may explain the failure of previous studies in detecting specific target zone nonlinearities: the existence of general type nonlinearities may distort possible target zone specific, S-shaped, nonlinearities rendering the previous studies unable to trace these specific nonlinearities.

This work can be extended in two ways. Firstly, this procedure is the first step towards developing further tests to capture specific types of nonlinearity such as the models of fads and noise trading behaviour, and the model of monetary policy strategies. Furthermore, tests allowing the co-existence and interaction of many types of nonlinearities can be developed. One way towards this would be to identify the specific functional form and the relevant variables to build a practical parametric nonlinear model for the purpose of out-of-sample forecasting, a hurdle proposed by Flood et al (1991). Secondly, the procedure for testing for a nonlinear relation, as proposed in this paper, could potentially be applied to test other theoretical nonlinearities such as the relationships between asset prices and fundamental variables.
REFERENCES


Bierens, H J (1997b) EasyReg, Dept of Economics, Pennsylvania State University, University Park, PA.


Kwiatkowski, D, P C B Phillips, P Schmidt, and Y Shin (1992) Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that time series have a unit root?, *Journal of Econometrics*, 54, 1-3, 159-178.


### TABLE 1
Unit root tests

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>LAGS</th>
<th>ADF - statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFR/DM</td>
<td>0</td>
<td>-1.867</td>
</tr>
<tr>
<td>ΔFFR/DM</td>
<td>0</td>
<td>-14.24</td>
</tr>
<tr>
<td>GUI/DM</td>
<td>7</td>
<td>-2.70</td>
</tr>
<tr>
<td>ΔGUI/DM</td>
<td>6</td>
<td>-9.52</td>
</tr>
<tr>
<td>GERMON</td>
<td>12</td>
<td>-0.65</td>
</tr>
<tr>
<td>ΔGERMON</td>
<td>11</td>
<td>-3.12</td>
</tr>
<tr>
<td>GERINC</td>
<td>1</td>
<td>-0.75</td>
</tr>
<tr>
<td>ΔGERINC</td>
<td>0</td>
<td>-22.48</td>
</tr>
<tr>
<td>NETMON</td>
<td>12</td>
<td>-0.82</td>
</tr>
<tr>
<td>ΔNETMON</td>
<td>11</td>
<td>-3.65</td>
</tr>
<tr>
<td>NETINC</td>
<td>2</td>
<td>-0.59</td>
</tr>
<tr>
<td>ΔNETINC</td>
<td>3</td>
<td>-10.97</td>
</tr>
<tr>
<td>FRAMON</td>
<td>0</td>
<td>-2.78</td>
</tr>
<tr>
<td>ΔFRAMON</td>
<td>12</td>
<td>-2.47</td>
</tr>
</tbody>
</table>

1. Notes:
2. 1. The definitions of variables: FFR/DM and GUI/DM are bilateral French Franc / Mark and Guild / Mark exchange rates, respectively. GERMON and GERINC are German money and industrial output, respectively. NETMON and NETINC are the Netherlands money and industrial output, respectively. FRAMON is the French money. All variables are in logarithms.

2. For the exchange rate variables (FFR/DM, GUI/DM), the ADF tests were conducted with a single dummy variable to capture the biggest single realignment in the central parity of these exchange rates.

3. In ADF tests, the number of lags is chosen using the Bayesian Information Criterion (BIC). The 5% critical value [from MacKinnon (1990)] is -2.88.

4. The Δ in front of the variable’s name denotes that the variable is in first difference.
### TABLE 2
Testing for linear cointegration using Johansen’s method

#### A. Netherlands - Germany

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>5% Critical Values</th>
<th>Test-statistics (15 lags in the VAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$H_A$</td>
<td>Max - L statistic</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$r = 1$</td>
<td>33.2</td>
</tr>
<tr>
<td>$r = 1$</td>
<td>$r = 2$</td>
<td>27.2</td>
</tr>
<tr>
<td>$r = 2$</td>
<td>$r = 3$</td>
<td>20.8</td>
</tr>
<tr>
<td>$r = 3$</td>
<td>$r = 4$</td>
<td>14.0</td>
</tr>
<tr>
<td>$r = 4$</td>
<td>$r = 5$</td>
<td>4.0</td>
</tr>
</tbody>
</table>

**Trace statistic**

| $r = 0$ | $r \geq 1$ | 68.9 | 68.3 |
| $r \leq 1$ | $r \geq 2$ | 47.2 | 41.7 |
| $r \leq 2$ | $r \geq 3$ | 29.5 | 20.9 |
| $r \leq 3$ | $r \geq 4$ | 15.2 | 7.4 |
| $r \leq 4$ | $r \geq 5$ | 4.0 | 0.1 |

**Conclusion**

$r = 0$

#### B. France - Germany

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>5% Critical Values</th>
<th>Test-statistics (15 lags in the VAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$H_A$</td>
<td>Max - L statistic</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$r = 1$</td>
<td>20.8</td>
</tr>
<tr>
<td>$r = 1$</td>
<td>$r = 2$</td>
<td>14.0</td>
</tr>
<tr>
<td>$r = 2$</td>
<td>$r = 3$</td>
<td>4.0</td>
</tr>
</tbody>
</table>

**Trace statistic**

| $r = 0$ | $r \geq 1$ | 29.5 | 41.2 |
| $r \leq 1$ | $r \geq 2$ | 15.2 | 18.2 |
| $r \leq 2$ | $r \geq 3$ | 4.0 | 0.01 |

**Conclusion**

$r = 2$

Likelihood ratio test for the exclusion of the exchange rate

$\chi^2(2) = 22.11$

(5% Critical Value = 5.99)

**Notes:**

1. For the Netherlands-Germany case, the variables included in the cointegration test are GUI/DM, the German money, the Netherlands money, the German output, and the Netherlands output. For the France-Germany case, the variables included in the test are FFr/DM, the French money and the German money.

2. $r$ denotes the number of cointegrating vectors. If $r = 0$, then there is no cointegration. Critical values (at 5%) are from Osterwald-Lenum (1992). If the computed statistic is below the critical value, then we cannot reject the $H_0$ hypothesis.

3. The number of lags in the corresponding VAR is set equal to 15. This number was established on the basis of likelihood ratio tests of alternative lag lengths.
### TABLE 3
Testing for nonparametric linear cointegration using the Bierens' method

#### A. Netherlands - Germany

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>5% Critical Region</th>
<th>Bieren's $\lambda_{min}$ test-statistic</th>
</tr>
</thead>
</table>
| $H_0$  
$r = 0$   
$r = 1$   | (0, 0.004)          | 0.000010                               |
| $H_0$  
$r = 1$   
$r = 2$   | (0, 0.005)          | 0.000010                               |
| $H_0$  
$r = 2$   
$r = 3$   | (0, 0.026)          | 0.007150                               |
| $H_0$  
$r = 3$   
$r = 4$   | (0, 0.075)          | 0.030160                               |
| $H_0$  
$r = 4$   
$r = 5$   | (0, 0.197)          | 0.10023                                |

**Conclusion**

\[ r = 0 \]

#### B. France - Germany

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>5% Critical Region</th>
<th>Bieren's $\lambda_{min}$ test-statistic</th>
</tr>
</thead>
</table>
| $H_0$  
$r = 0$   
$r = 1$   | (0, 0.008)          | 0.00025                                |
| $H_0$  
$r = 1$   
$r = 2$   | (0, 0.017)          | 0.000371                               |
| $H_0$  
$r = 2$   
$r = 3$   | (0, 0.111)          | 0.2815                                 |

**Conclusion**

\[ r = 2 \]

**Notes:**

1. The critical values are from Bierens (1997b). If the computed statistic is within the critical region, then we reject the $H_0$. However, if all the null hypotheses are rejected, then the conclusion is no cointegration ($r=0$) (Bierens, 1997b).
TABLE 4
Unit root tests and bootstrapped critical values for the ACE-transformed variables for Germany and the Netherlands

<table>
<thead>
<tr>
<th>Variables</th>
<th>ADF -statistic</th>
<th>5% Bootstrapped Critical Value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>GUIDM</td>
<td>-3.53</td>
<td>-2.88</td>
<td>I(0)</td>
</tr>
<tr>
<td>GERMON</td>
<td>-4.20</td>
<td>-2.79</td>
<td>I(0)</td>
</tr>
<tr>
<td>GERINC</td>
<td>-0.84</td>
<td>-2.88</td>
<td>I(1)</td>
</tr>
<tr>
<td>ΔGERINC</td>
<td>-5.88</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>NETMON</td>
<td>-2.42</td>
<td>-2.28</td>
<td>I(0)</td>
</tr>
<tr>
<td>NETINC</td>
<td>-0.060</td>
<td>-2.81</td>
<td>I(1)</td>
</tr>
<tr>
<td>ΔNETINC</td>
<td>-7.59</td>
<td>----</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Bootstrapping is repeated 1000 times.
2. The ADF statistics are calculated using 5 lags, based on the BIC criterion.
3. See Note 3 in Table 1 for definitions of variables.
TABLE 5  
Testing for nonlinear cointegration using the Bierens’ and Johansen’s methods for the original and ACE-transformed variables: Netherlands and Germany

A. Bierens’ method

<table>
<thead>
<tr>
<th>H₀</th>
<th>H₁</th>
<th>Critical Regions at the 5% level</th>
<th>Bierens’ λₘᵣₐᵢₙ test -statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 0</td>
<td>r = 1</td>
<td>(0, 0.004)</td>
<td>0.00001</td>
</tr>
<tr>
<td>r = 1</td>
<td>r = 2</td>
<td>(0, 0.005)</td>
<td>0.00001</td>
</tr>
<tr>
<td>r = 2</td>
<td>r = 3</td>
<td>(0, 0.026)</td>
<td>0.01015</td>
</tr>
<tr>
<td>r = 3</td>
<td>r = 4</td>
<td>(0, 0.075)</td>
<td>0.77906</td>
</tr>
<tr>
<td>r = 4</td>
<td>r = 5</td>
<td>(0, 0.197)</td>
<td>3.5884</td>
</tr>
</tbody>
</table>

Conclusion  
Test for exclusion of the exchange rate  
L-Max stat. = 2622  
Trace stat. = 2624  
Critical Value = 4.84  
(at the 5% level)

B. Johansen’s method

<table>
<thead>
<tr>
<th>H₀</th>
<th>H₁</th>
<th>Original 5% Critical Values</th>
<th>Bootstrapped 5% Critical Values</th>
<th>Test-statistics (15 lags in the VAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>L-Max statistic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r = 0</td>
<td>r = 1</td>
<td>33.2</td>
<td>52.7</td>
<td>26.5</td>
</tr>
<tr>
<td>r = 1</td>
<td>r = 2</td>
<td>27.2</td>
<td>34.1</td>
<td>19.6</td>
</tr>
<tr>
<td>r = 2</td>
<td>r = 3</td>
<td>20.8</td>
<td>23.1</td>
<td>13.4</td>
</tr>
<tr>
<td>r = 3</td>
<td>r = 4</td>
<td>14.0</td>
<td>15.6</td>
<td>7.3</td>
</tr>
<tr>
<td>r = 4</td>
<td>r = 5</td>
<td>4.0</td>
<td>9.9</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Trace statistic  
| r = 0 | r ≥ 1 | 68.9 | 123.6 | 67.1 |
| r ≤ 1 | r ≥ 2 | 47.2 | 76.3  | 40.6 |
| r ≤ 2 | r ≥ 3 | 29.5 | 45.5  | 20.9 |
| r ≤ 3 | r ≥ 4 | 15.2 | 24.3  | 7.6  |
| r ≤ 4 | r ≥ 5 | 4.0  | 9.9   | 0.2  |

Conclusion  
| r = 0 |

Notes :
1. The five variables included in the cointegration tests are the original GUI/DM, the original German money, the original Netherlands money, the ACE-transformed German output and the Netherlands ACE-transformed output.

2. The critical values are from Bierens’ (1997). If the computed statistic is within the critical region, then we reject the H₀.

3. The number of lags in the corresponding VAR is set equal to 15. This number was established on the basis of likelihood ratio tests of alternative lag lengths.
**TABLE 6**
Nonlinear nonparametric Granger causality test results for France - Germany

<table>
<thead>
<tr>
<th>Lx = Ly</th>
<th>CS</th>
<th>Test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.029</td>
<td>1.905 *</td>
</tr>
<tr>
<td>9</td>
<td>0.0288</td>
<td>1.721 *</td>
</tr>
<tr>
<td>10</td>
<td>0.0333</td>
<td>1.851 *</td>
</tr>
<tr>
<td>11</td>
<td>0.0386</td>
<td>1.701 *</td>
</tr>
<tr>
<td>12</td>
<td>0.0487</td>
<td>1.679 *</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lx = Ly</th>
<th>CS</th>
<th>Test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.0006</td>
<td>0.027</td>
</tr>
<tr>
<td>9</td>
<td>0.0083</td>
<td>0.420</td>
</tr>
<tr>
<td>10</td>
<td>-0.0040</td>
<td>-0.165</td>
</tr>
<tr>
<td>11</td>
<td>-0.0091</td>
<td>-0.259</td>
</tr>
<tr>
<td>12</td>
<td>0.0149</td>
<td>0.448</td>
</tr>
</tbody>
</table>

**Notes:**

1. This Table reports the results of the modified Back and Brock test for nonlinear Granger causality.

2. Lx = Ly denotes the number of lags on the residuals series used in the modified Back and Brock test. In all cases, the value of the length scale, e, is set to 1.6.

3. CS denotes the difference between the two conditional probabilities in equation (3).

4. The modified Back and Brock test statistic is the standardised statistic in equation (4).

5. Under the null hypothesis on nonlinear Granger noncausality, the test is asymptotically distributed N(0,1).

6. The asterisk * denotes that the statistic is significant at the 5% level, and the H0 hypothesis of no (nonlinear) causality is rejected.

7. Number of observations = 187.
TABLE 7
Testing for bubbles

<table>
<thead>
<tr>
<th>Variables</th>
<th>ARFIMA (p,d,q)</th>
<th>d</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFR/DM</td>
<td>(12, d, 0)</td>
<td>0.980 (0.24)</td>
<td>I(1)</td>
</tr>
<tr>
<td>GUI/DM</td>
<td>(12, d, 0)</td>
<td>1.180 (0.24)</td>
<td>I(1)</td>
</tr>
<tr>
<td>GERMON</td>
<td>(48, d, 0)</td>
<td>1.017 (0.26)</td>
<td>I(1)</td>
</tr>
<tr>
<td>GERINC</td>
<td>(3, d, 0)</td>
<td>1.067 (0.23)</td>
<td>I(1)</td>
</tr>
<tr>
<td>NETMON</td>
<td>(24, d, 0)</td>
<td>1.08 (0.24)</td>
<td>I(1)</td>
</tr>
<tr>
<td>NETINC</td>
<td>(6, d, 0)</td>
<td>1.185 (0.24)</td>
<td>I(1)</td>
</tr>
<tr>
<td>FRAMON</td>
<td>(36, d, 0)</td>
<td>1.13 (0.24)</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

Notes:
1. p, d, q are respectively the orders of autoregression, integration and moving average components of the time series.
2. Standard errors are in parentheses below the d values.
3. See Note 1 in Table 1 for the definition of variables.